

# **Empleo de aplicaciones tecnológicas en el tratamiento de temas de Probabilidad y Estadística. Dificultades presentadas por los estudiantes en la formulación de planteamientos correctos**

*Use of technological applications in the treatment of Probability and Statistics subjects. Difficulties presented by students in formulating correct approaches*

*Uso de aplicações tecnológicas no tratamento de problemas de Probabilidade e Estatística. Dificuldades apresentadas pelos alunos na formulação de abordagens corretas*

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## **Resumen**

En la presente investigación se aborda la problemática de la dificultad que presentan los estudiantes en el planteamiento de soluciones adecuadas a problemas que comúnmente se emplean en la materia de Probabilidad y Estadística en el nivel superior. Se encontró que tanto las diversas temáticas abordadas en los problemas de probabilidad, las cuales no les son familiares a los estudiantes, además del lenguaje que es empleado en los textos, les causa conflicto y una inadecuada interpretación de los problemas, lo que los conduce a proponer un planteamiento erróneo en sus soluciones.

El marco conceptual que soporta este estudio con respecto a la relación que existe entre el lenguaje y las matemáticas proviene de las ideas de Chomsky que subrayan la importancia de esclarecer aspectos semánticos en los problemas de matemáticas, así como las de Duval, quien pone énfasis en las representaciones y sus implicaciones para el estudio

de los procesos de enseñanza y aprendizaje de las matemáticas. Con respecto a su desarrollo, este estudio tiene un enfoque social constructivista debido a los contextos que abordan los problemas con los que se trabajan.

Se empleó una metodología experimental al trabajar con un grupo de 32 estudiantes quienes cursaban la materia de Probabilidad y Estadística en su tercer semestre de la carrera de Ingeniería en Sistemas Computacionales. Durante el proceso de enseñanza y aprendizaje de dicha materia, se propuso el empleo de diferentes plataformas y aplicaciones tecnológicas para tener un acercamiento diferente al usual.

Tanto los estudiantes como los docentes que participaron en el estudio fueron notificados del trabajo que se realizaría, respetando su decisión en la participación de este.

Primero, se aplicó un cuestionario para conocer su percepción en relación con el uso de la tecnología; después y durante un mes, se trabajó con el grupo con distintas aplicaciones tecnológicas; y finalmente, se aplicó un cuestionario de evaluación.

Los resultados indicaron que uno de los recursos o estrategias que ayudan en la comprensión de los problemas es el uso de material videográfico que represente las situaciones planteadas, permitiendo al estudiante visualizar diversos aspectos de los problemas y no solo trabajar en el terreno algebraico y con el uso exclusivo de fórmulas. Además, la formulación de varias preguntas condujo al estudiante tanto a la determinación de las variables involucradas, como a aclarar lo que se está solicitando medir, evaluar o encontrar en un problema.

**Palabras clave:** aplicaciones tecnológicas, distribuciones de probabilidad, educación, nivel superior, probabilidad condicional.

## **Abstract**

The present research is focused in the difficulty that the students show in the approach of adequate solutions to problems that are commonly used in the field of Probability and Statistics at the College. It was found that both the various issues addressed in the problems of probability, the cells are not familiar to students, in addition to the language that is commonly used in the texts, causes conflict to propose an appropriate approach in their solutions.

The conceptual framework that supports the study in relation to the link between language and mathematics are the ideas of Chomsky on the importance of clarifying semantic aspects in the problems of mathematics and those of Duval who emphasizes the representations and their implications for the study of the teaching and learning processes of mathematics. In relation to the development of the study, it has a constructivist social approach due to the contexts that address the problems with which they work.

Both the students and the teachers who participated in the study were notified of the work that would be done, respecting their decision in the participation of the same.

The experimental methodology was used when working with a group of 32 students who studied the subject of Probability and Statistics in the third semester of the field of Engineering in Computer Systems. First a diagnostic questionnaire was applied to know its perception in relation to the use of technology, after and during a month was worked with the group different applications and finally an evaluation questionnaire was applied. The results indicated that some resources or strategies that help in the understanding of the problems, is the use of videographic material that represents the presented situations, allowing the student to visualize various aspects of the problems and not only work in the algebraic terrain and with the exclusive use of formulas. In addition the formulation of several questions that lead the student both to the determination of the variables involved, to clarify what is being asked to measure, evaluate or find in a problem.

**Keywords:** technological applications, probability distributions, education, college, conditional probability.

## Resumo

Na presente investigação, aborda-se o problema da dificuldade que os alunos apresentam na abordagem de soluções adequadas aos problemas que são comumente usados na questão de Probabilidade e Estatística no nível superior. Verificou-se que ambas as várias questões abordadas nos problemas de probabilidade, que não são familiares para os alunos, além da linguagem que é usada nos textos, provocam conflitos e uma interpretação inadequada dos problemas, o que os leva a propor uma abordagem errada em suas soluções.

O quadro conceitual que apóia esse estudo em relação à relação entre linguagem e matemática vem das idéias de Chomsky que enfatizam a importância de esclarecer aspectos semânticos em problemas de matemática, bem como os de Duval, que enfatiza as representações e suas implicações para o estudo dos processos de ensino e aprendizagem de matemática. Com respeito ao seu desenvolvimento, este estudo tem uma abordagem social construtivista devido aos contextos que abordam os problemas com os quais eles trabalham. Uma metodologia experimental foi utilizada ao trabalhar com um grupo de 32 alunos que estudaram o tema de Probabilidade e Estatística em seu terceiro semestre do curso de Engenharia de Sistemas Informáticos. Durante o processo de ensino e aprendizagem deste assunto, o uso de diferentes plataformas e aplicações tecnológicas foi proposto para ter uma abordagem diferente da usual.

Estudantes e professores que participaram do estudo foram notificados do trabalho que seria feito, respeitando sua decisão na participação deste.

Em primeiro lugar, foi aplicado um questionário para conhecer sua percepção em relação ao uso da tecnologia; Depois e durante um mês, trabalhamos com o grupo com diferentes aplicações tecnológicas; e, finalmente, foi aplicado um questionário de avaliação.

Os resultados indicaram que um dos recursos ou estratégias que ajudam na compreensão dos problemas é o uso de material videográfico que representa as situações colocadas, permitindo que o aluno visualize vários aspectos dos problemas e não apenas trabalhe no campo algébrico e com o uso exclusivo de fórmulas. Além disso, a formulação de várias questões levou o aluno à determinação das variáveis envolvidas e a esclarecer o que está sendo pedido para medir, avaliar ou encontrar em um problema.

**Palabras-chave:** aplicações tecnológicas, distribuições de probabilidade, educação, nível superior, probabilidade condicional.

**Fecha Recepción:** Abril 2017

**Fecha Aceptación:** Diciembre 2017

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## Introduction

In the matter of Probability and Statistics at a higher level, the approach of the problems or exercises is usually on very varied topics and some of these are very extensive (Batanero and Sánchez, 2005). Due to this, the premise that is handled in this research is that the student makes mistakes in the interpretation or understanding of the problem, which leads to an erroneous result. Likewise, it presupposes that with the help of graphic material, digital animations, as well as computer programs, the student can have an approach that leads him to correctly interpret the problem and reach his solution.

Cai (2003, p.245) and Lester and Charles (2003, p.129) comment that in order to help students solve problems successfully it is necessary to make them aware that resolution skills often develop slowly, requiring sustained long-term attention. term and thereby make them see which problem solving is an integral part of the mathematics program. In addition, teachers must develop a culture in the classroom that helps make problem solving a regular, consistent, and practical activity. Students should also understand the importance of participating regularly in challenging exercises.

To solve a problem requires undertaking a process that starts with the interpretation of the mathematical text; likewise, there are several iterative moments where tests are carried out, data are ordered, these are combined, mathematical concepts are used and the information is verified, thus determining the response (Lesh and Zawojewski, 2007). Polya (1985, pp. 5-6), pointed out that every process of solving a problem requires going through four stages: understanding the problem, designing a plan, executing it and verifying what has

been obtained. Later, Shoenfeld (1992, p.339) stated that each stage of those established by Polya should be divided in turn, in order to be more thorough in employing different strategies (heuristics and metaheuristics) that would lead to the solution of the problem.

That is why, in the present investigation, the use of elements, graphic material and computational tools was proposed throughout the process involved in solving the problems of the Probability and Statistics subject. This proposal was implemented in a group of 32 students belonging to the Higher School of Computing of the National Polytechnic Institute [ESCOM-IPN] (2009), taking as reference the textbooks indicated in the study program, with the objective that the student will achieve a continuous improvement in the reading comprehension of problems related to the learning unit in question.

## **State of the art**

Scientific research on the relationship between mathematics and language has shown in a general way that it is intrinsic both in its learning and in its application by students, as well as being substantial and taking several decades in the pedagogical field.

Monroe (1996) proposes the realization of an analysis between the relationship between language and mathematics to establish new teaching strategies. For this, the focus is on the application of mathematics to everyday life, based on the premise that the student acquires solid knowledge and also has a favorable perception of such science.

Several studies have identified language as one of the most important processes to learn mathematics and to solve problems, among them the one made by Filio (2005), who refers to the theoretical study of Chomsky, stating that before presenting a mathematical content it requires defining exactly what is the function of language in mathematics and how it influences its development as a science; at the same time it places language and mathematics as intermediaries for the development of thought. Another important part of the investigation carried out by Filio has to do with the analysis of the language of some areas of mathematics, as well as the use of a language more comprehensible by its form, far from the mathematical meaning.

On the other hand, García (2014), who works with university students, considers the relationship between language and mathematics as a communication problem, although, he clarifies, it is not unique; he states that it is the main route between the teacher and the student, with the first subject having questions about the problems that are suspected when teaching the class. García (Ibidem) points out that the mistakes that students make in solving math problems are not only reduced to incorrect answers, but that many components are also present, among them the incorrect interpretation of the language. In this regard, Rico (1995) comments that an error occurs if a mathematical situation is translated incorrectly from one symbol language to another. Share this statement Guevara (2007), noting that in the act of reading a text is not only deciphering a code, but also is giving meaning and meaning to it.

García (2014) He adds that in a general way students consider mathematics as memorizing a series of formulas and performing several numerical operations. He also mentions that this way of thinking affects mathematical communication, since there is a gap between the student's language and the organized language of mathematics.

For his part, Duval (2004) notes that the understanding of texts in mathematics is considered a cognitive activity, so it is necessary to use different records of representation, such as algebraic, numerical and graph, which are different from the common language .

D'Amore (2006) comments that to explain some concepts teachers regularly use a "school language" and sometimes fail to clarify the understanding of the situation, because when working in mathematics a particular syntax is addressed.

It can be said that each of the mathematics uses a particular form of language and symbols and that most of the time they are different depending on the context or its usefulness.

Regarding probability and learning, Batanero and Sánchez (2005) make reference to some key objectives for students of this branch of knowledge, which, in addition, they consider valuable in a professional field:

- Be clear about the idea of data variability and be able to summarize it applying the theory of probability.
- Normal distributions as useful models even when they are rarely perfect.
- Make valid inferences from previous measurements.



- The correlation between two variables does not imply cause and effect.
- Statistics can prove and lead to conclusions close to reality, and, although they may suggest facts, this must be accepted as irrefutable.

Some of the main problems for an effective learning of the subject of Probability and Statistics are due to problems of perception (Batanero and Sánchez, *Ibidem*), which are related to concepts such as:

- *Representativeness*: from a small sample (or examples) it tends to generalize for a much larger sample, an error of many students and people in general.
- *Conjunction fallacy*: believe that two events are more likely than one (without being dependent), even if it is false in most cases.

Thus, Batanero and Sánchez (*Ibidem*) based on articles on the same subject, find that many of these have in common the conclusion that there is inappropriate reasoning about some ideas about statistics and probabilistic extended, persistent and similar at all ages.

On the other hand, Lesh and Zawojewski (2007) point out that in the resolution of problems one should not lose sight of the mathematical interpretation of the situations that are proposed, because many times students do not use a mathematical thought, but rather involve aspects of your daily experience

The research developed and presented in this document is also based on the constructivist social approach to science learning (Sierpinska and Lerman, 1996, Prados and Cubero, 2005, Pons and Serrano, 2011), where the student is expected to build the meanings based on their previous knowledge and through a contextualized activity or problem, as well as being able to work collectively.

Precisely the problems that arise in the matter of Probability and Statistics correspond to very varied contexts, in such a way that the proposal of the professor on the analysis that is made in class can be of collective form.



## **Study problem**

There are several factors why students fail to obtain a good performance in the Probability and Statistics learning unit at ESCOM-IPN. In the state of art reference is made to the importance of the relationship between language and mathematics and that, due to the inadequate understanding of the texts, mathematical mistakes fall. The factor that in this investigation was considered was related to the issues that address the problems, leaving the approach of the problem as follows: the great variety and number of topics, not related to the student and addressed in the problems of the unit Probability and Statistics learning of most textbooks (Walpole, Myers and Ye, 2012; Meyer, 1999), is one of the many causes that lead the student to make mistakes and not to successfully face the problems proposed in class.

For all the above, in this research is intended to help students and teachers in the understanding and interpretation of mathematical texts, specifically texts of Probability and Statistics. And thus contribute to students to better solve the exercises of that subject and link the content of these situations to the social world in which they are, since education in the classroom is by nature of a social nature, that is, its function is that students have a learning that allows both personal and social development (Pons and Serrano, 2011, Punset 2011).

In order to show the result of the event being analyzed, have a better understanding of it and go beyond a number without further interpretation, it is proposed to use didactic and graphic tools such as applications and programs.

## **Methodology**

The methodology used was of an experimental nature, since first an initial questionnaire was designed and applied to the students who were studying during the morning shift the subject of Probability and Statistics of one of the academic units of the ECOM-IPN, corresponding to the cycle school year 2017/2, that is, corresponding to the January-June 2017 cycle (261 students). Once the answers were obtained, the coincidences with the literature consulted were analyzed and reviewed.

At the same time, four teachers were interviewed who were teaching the aforementioned subject. The sample of teachers was selected at random, since the seven

teachers who attended the subject in the school year specified above were invited and only four teachers accepted to participate.

Subsequently, an inquiry was made and the computational tools were tested in relation to the topics covered by the curricular program of the academic unit where the study was made (ESCOM-IPN, 2009). We worked with a sample of 32 students that were part of one of the 8 groups that are held in the ESCOM-IPN, where these are formed in a random manner. The students were studying Probability in the third semester of the Computer Systems Engineering career: young people between 20 and 23 years old. These students had already taken courses in Calculus, Discrete Mathematics and Differential Equations, which implies that they had the tools offered by these subjects. In these previous courses they had solved problems in contexts of physics, as well as those of mathematics (geometry, trigonometry, analytical geometry) and other areas such as electronics and communications.

The students were asked to work for a month with some technological applications, such as software and web pages (Khan Academy, Educatina, Probability Distributions), which are described below. The topics that were addressed were the conditional probability and probability distributions.

After its use, an evaluation questionnaire was applied, but this time only the 32 students who worked with the computer applications participated. In this part the interest was focused on evaluating the utility that they had found with the use of these.

## **Methodological instruments used**

### **Initial questionnaire addressed to students**

The use of didactic and graphic tools such as programs and applications helps the student to consider what he / she would have to solve around probability. To detect which are the main difficulties that have to solve and understand a problem of this nature, as already mentioned above, an initial questionnaire was applied to ESCOM-IPN students who studied the subject of Probability and Statistics, who were enrolled in the third semester of the school year January-June 2017, which were a total of 261 students. In Table 1, this initial survey is shown.

**Tabla 1.** Preguntas del cuestionario inicial para los estudiantes.

1. ¿Qué tan difícil es para ti la probabilidad?	7. ¿Qué método utilizarías para poder resolver los ejercicios de esta asignatura?
2. ¿Consideras que el aprendizaje depende más de la enseñanza del profesor que del desempeño del alumno?	8. ¿Consideras que sería importante ocupar una parte del curso para analizar la interpretación de los problemas? Sí / No ¿Por qué?
3. ¿Cuál crees que sea la principal causa por la que no se logra comprender el curso de Probabilidad y Estadística?	9. ¿Crees que la probabilidad está implícita en hechos de la vida cotidiana? Sí / No ¿Por qué?
4. ¿Cómo lograrías mejorar la parte de entendimiento del curso?	10. ¿Opinas que ocupar material didáctico y gráfico ayude a entender mejor el planteamiento del problema?
5. ¿Qué dificultades presentas al resolver un ejercicio?	
6. Personalmente, ¿qué método de enseñanza te resulta más eficiente para entender la probabilidad y estadística?	

Fuente: Elaboración propia.

### Way of working with the computational applications

For the implementation of computational tools as assistants in solving problems, the class was divided into four stages:

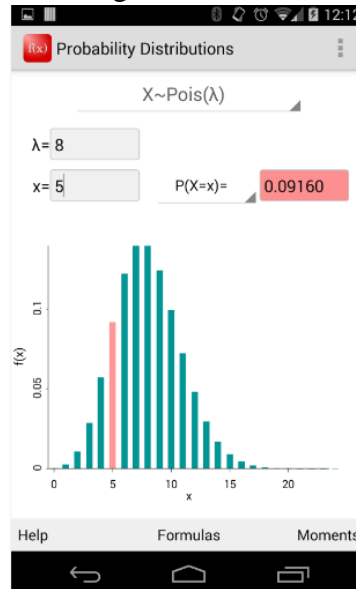
1. *Start:* the teacher proposed a problem of the conditional probability issue or of some probability distribution, whether discrete or continuous (binomial, exponential, geometric or normal). He used the Khan Academy platform to show a video related to the problem.
2. *Development:* the teacher asked a series of questions about the problem presented. Once the student recognized the probability distribution that could be used to solve the problem, he used the Statistics Calculator software or the Probability Distributions software to perform the calculations.

3. *Evaluation*: in this part of the class the teacher used the Khan Academy platform to present the student problems of the same subject and gave him time to work on that platform. In it, students could review videos where solution processes were shown, as well as giving advice on how to solve problems. Also on the same platform the problems and exercises solved by the students were rated.
4. *Feedback*: the teacher invited the students to explain some of the problems reviewed and resolved any doubts they might have.

### **Computational Applications**

Probability Distributions: is an app available in Android for probability distributions, such as binomial, geometric, Poisson, hypergeometric distribution and negative binomial distribution, as well as computation probabilities, determine percentiles, function argument of probability density for normal (Gaussian), t, chi-square, F, exponential, gamma, beta and normal distributions of record Matthew Bognar (2017).

This app was very helpful because it reduced the search time of the required data. First the variables of the problems were detected, with this the possible distributions that could be used were analyzed and, finally, the one that was most suitable was determined. Then the values were entered in the corresponding boxes and the application showed the corresponding answers. It was a great support, since it showed the graphs and not only the numerical results, which allowed an interpretation of what was obtained. In figure 1 the image of one of the interfaces is shown.

**Figura 1.** Interfaz gráfica de Probability Statics.

Fuente: Matthew Bognar, 2017.

*Statistics Quick Reference*: application available for Android with free or paid version (Statistics Quick Reference, 2017). The application has a small introductory dictionary that includes the basic concepts and definitions to become familiar with the terminology of the subject. Each of the topics to be explained is followed by a numerical example in order to establish the knowledge and ensure the comprehension of the definition. (Fig. 2). This application contributed ideas for the understanding of the problems worked on.

*Educatina*: this web page has 8 sections that in turn each have several subtopics, depending on the selected subject. In the Probability and Statistics section there is a wide index of topics. It is a didactic page that helps the student to understand the topics, reinforcing the understanding with exercises. In addition, you have the option of qualifying said exercises and it shows if the selected answer is erroneous and, in such case, throws the correct one.

**Figura 2.** Interfaz gráfica de Quick References.

How to read Tables!

(a) How to read the Standard Normal Distribution Table!

Pre-requisites:

- The standard normal distribution table
- Z score

Example 1:

Find the value of  $P(Z \leq 1.41)$

z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628
0.5	0.6915	0.6950	0.6985
0.6	0.7257	0.7291	0.7324
0.7	0.7580	0.7611	0.7642
0.8	0.7881	0.7910	0.7939
0.9	0.8159	0.8186	0.8212
1.0	0.8413	0.8438	0.8461
1.1	0.8643	0.8665	0.8686
1.2	0.8849	0.8869	0.8888
1.3	0.9032	0.9049	0.9066
1.4	0.9192	0.9207	0.9222

Fuente: Statistics Quick References, 2017.

In this application the student found the explanation to several problems that were presented to him, because graphics are used to represent what is being formulated in the problem, as well as questions that help to shred the exercise into parts.

Statistics Calculator: This app is available on Android and has 8 distributions: Binomial, Normal, Uniform, Poisson, Exponential, Gamma, Geometric, and Joint Distribution. In addition to calculating the probabilities, calculate the covariance and correlation for the joint distribution, including mean, standard deviation and variance for all distributions (Statistics Calculator, 2017). This application was used with the students to calculate the values of the distributions that would allow solving the problems raised.

Khan Academy: It is a web page where there are three sections, mainly: student, teacher and parents. Here are several topics of study, specifically talking about probability and statistics. It also contains an index of topics, where, through a video, each of them is explained. Subsequently has a section of exercises related to the subject previously seen. It has the option of being able to qualify the result showing if this is correct or not (Khan Academy, 2017).

This platform was used in the classes in the following way. First, a problem was presented to the student and a video was shown explaining what was asked to solve (this helped the students a lot because, as it has been described throughout the present article, it is the weakest point that the majority of those who integrate it). Later problems with other variables were shown and the student was asked to identify them, as well as to identify what was being asked to determine. To this end, the formulation of questions related to different aspects of the problem and the response to each of them was resorted to. Finally the student solved different exercises and the same system evaluated it.

Figure 3 shows the captured image of one of the videos that contains the Khan Academy platform where a problem is presented and by underlining some words it shows the student what is being asked to determine. Then show a solution design and follow it step by step until you get to the answer.

**Figura 3.** Ejemplo del proceso de solución de un problema.

Ejemplo acertando dos preguntas en un examen

En un examen de opción múltiple, la pregunta 1 tiene 4 opciones y la pregunta 2 tiene 3 opciones. Cada pregunta sólo tiene una respuesta correcta. ¿Cuál es la probabilidad de contestar ambas preguntas bien si se elige la respuesta de modo aleatorio?

$P(\text{bien en \#1})$  y  $P(\text{bien en \#2})$  son eventos independientes

$$P(\text{bien en \#1 y \#2}) = P(\text{bien en \#1}) \cdot P(\text{bien en \#2})$$

$$\frac{1}{12} = \frac{1}{4} \cdot \frac{1}{3}$$

		Pregunta #1			
		i1	i2	i3	c
Pregunta #2	i1				
	i2				
	c				

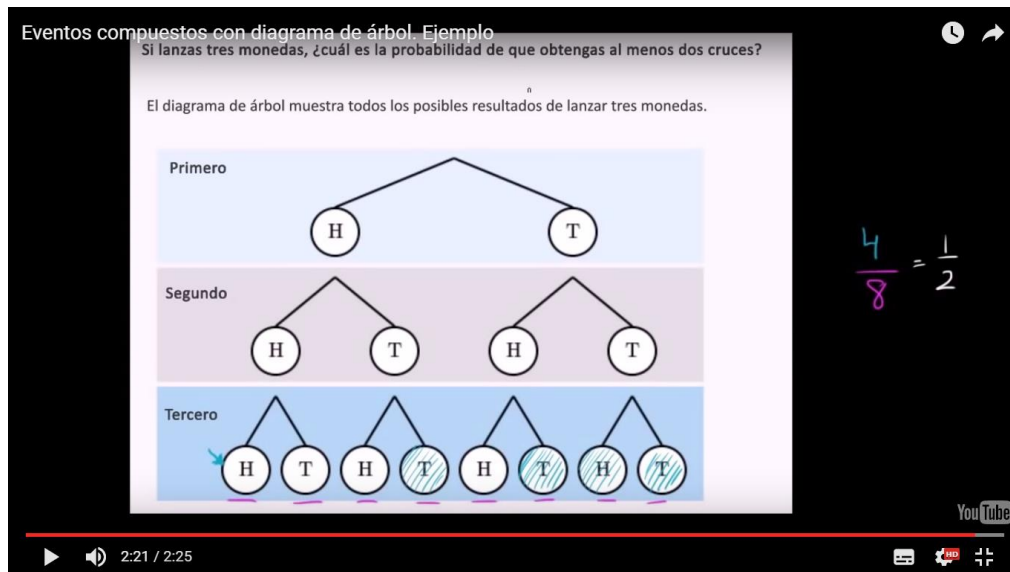
$\frac{1}{12}$

Fuente: Khan Academy, 2017.

Figure 4 shows another problem and the graphic material that was used to describe step by step, both the interpretation of the problem text and the entire solution process.



**Figura 4.** Ejemplo del proceso de solución de un problema.



Fuente: Khan Academy, 2017.

## Results and analysis

### Results of the initial questionnaire

Derived from the application of the initial questionnaire addressed to the students, the responses were organized into categories for analysis. These categories are shown in table 2.

**Tabla 2.** Organización de la información de las respuestas en categorías.

Pregunta	Categorías	Porcentaje
¿Qué tan difícil es para ti la probabilidad?	Nivel de dificultad: Alto Intermedio Bajo	70% 30% -
¿Consideras que el aprendizaje depende más de la enseñanza del profesor que del desempeño del alumno?	Más del profesor que del alumno Igual Más del alumno que del profesor	80% 10% 10%
¿Cuál crees que sea la principal causa por la que no se logra comprender el curso de Probabilidad y Estadística?	Interpretación errónea de los textos de los problemas. Planteamiento deficiente	90% 80%
¿Cómo lograrías mejorar la parte de entendimiento del curso?	Presentar ejemplos prácticos. Analizar los problemas para formular planteamientos correctos	70% 90%
Personalmente, ¿qué método de enseñanza te resulta más eficiente para entender la probabilidad y estadística?	Desglosar en varias preguntas la pregunta principal del problema. Utilizar material gráfico que represente el contenido del problema Más participación del alumno en clase	48% 38% 80%
¿Qué método utilizarías para poder resolver los ejercicios de esta asignatura?	Analizar los problemas para formular planteamientos correctos. Desglosar en varias preguntas la pregunta principal del problema.	90% 48%
¿Consideras que sería importante ocupar una parte del curso para analizar la interpretación de los problemas? Si/No ¿Por qué?	Sí, porque es la base para su solución No	100% -
¿Crees que la probabilidad está implícita en hechos de la vida cotidiana? Si/No ¿Por qué?	Sí En juegos Aspectos meteorológicos Toma de decisiones	100%
¿Opinas que ocupar material didáctico y gráfico ayude a entender mejor el planteamiento del problema?	Sí No	100% -

Fuente: Elaboración propia

90% of the students agree with the implementation of the proposal on the use of technology as a tool in the learning of this subject, in addition to being aware that a problem with which the majority faces is the erroneous interpretation and understanding of exercises, whose writing seems long and confusing. However, it was also found that students prefer different teaching methods, without predominating one in particular.

Eighty percent of the students mentioned that a good teaching base on the part of the teacher is necessary for a good learning of the subject. Although the student has the obligation to review the topics seen in class, the teacher has an important part in the development and understanding of the problems. This is because if the instructor is awarded the entire teaching process and if it fails to explain in a concrete and correct manner how is the procedure to be followed, the student will have an erroneous or incomplete idea that will result in incomplete exercises, misassigned data or lack of compression capacity and development of the problem. But if the teacher shares the teaching process by allowing the student to discover, investigate and be a participant in the class, then the students will have the responsibility to find options and share them with their classmates and the teacher and thus address the problems and the solution processes. In this search, both (teacher and students) have the opportunity to review different technological applications and platforms that have material or resources to support this task.

In this last point, the methods of education preferred by the students were the following: 38% percent of the students agree that the best teaching method they could have is through graphic exercises, because they have a vision and greater magnitudes of the problem. While 14% agree that the least efficient way to understand a problem is solving exercises by means of statements, because in many cases these are written in a confusing and long way, so, in the process of data collection, the student tends to lose sight of the general objective of the exercise, causing him to read the problem more than once to understand where he wants to go and collect the correct information.

On the other hand, 90% of the students surveyed agree that it is important to dedicate a considerable amount of time during the course to the interpretation and analysis of each problem, in order to have a better knowledge regarding the emergence of said data and the meaning of the result obtained, since the probability is implicit in facts of everyday life,

although not always directly, as it is in making a decision to win a flip, in board games or games of lottery and in many other areas that are developed on a day-to-day basis.

Concluding, the vast majority of students agree that the use of graphic tools would facilitate the improvement in the understanding of the subject. They also agree that it depends a lot on the way in which the teacher opens the panorama to the student regarding the resolution of the problems and their interpretation.

### **Interview with teachers**

Four of the professors who teach the subject were interviewed in the school where the study was conducted, they were asked to be recorded, then the information provided in the open questions of the interview was ordered through the establishment of categories, which were used for its analysis. This information is presented in table 3.

**Tabla 3.** Preguntas de la entrevista realizada a los profesores y concentración de resultados.

Pregunta	Categorías	Porcentaje
¿Considera que el curso de probabilidad y estadística es complejo? Sí / No ¿Por qué?	Por la cantidad de temas que tiene. Por el tipo de problemas a resolver. Por el poco tiempo con el que se cuenta para impartir todo el temario. Por la complejidad matemática de los temas.	100% 100% 100% 75%
¿Dónde fallan los alumnos con más frecuencia al ver los temas?	En la interpretación de los textos de los problemas. En la selección de la distribución adecuada, es decir, decidir si corresponde a una distribución binomial o a una geométrica negativa. En la selección correcta de la fórmula. En el planteamiento correcto del problema.	100% 75% 50% 100%
¿Qué método de enseñanza consideras que es el más apto para que el alumno entienda bien los temas?	Resolución de problemas en clase. Leer más de una vez los problemas. Determinar las variables del problema y sus relaciones.	100% 100% 75%
¿Utilizas alguna herramienta como apoyo para que el alumno entienda mejor los temas? Sí / No ¿Cuál? ¿Por qué?	Calculadora. Libros de texto. No usan aplicaciones tecnológicas.	100% 100% 100%
¿Cuál consideras que es el principal problema por el cual los alumnos no comprenden bien los temas?	La falta de comprensión de los textos de los problemas. No tienen un buen planteamiento. No ponen atención en las clases.	100% 100% 50%

Fuente: Elaboración propia.

As a result of the interviews conducted with the professors, it was found that most of them consider that the Probability and Statistics course has an extensive syllabus to cover it completely during a semester. They also mentioned that a common problem in students is to make mistakes in the approach of the problem, due to a misinterpretation that, according to the teachers surveyed, is due to the lack of attention during the class, in addition to the many topics that must impart, not forgetting that some of the problems are worded in a confusing way.

The teachers to whom the questionnaire was applied proposed a series of measures to reduce the incidence of this problem:

- Read the problem more than once.
- Underline key words that could indicate the need to do a certain process.
- Support of probability tables, of applications that serve to corroborate obtained results and of books of the learning unit with their exercises.
- Solve lists of exercises for each topic.
- Attend counseling when a topic is not completely understood.

In the results, in addition, it is highlighted that 60% of the respondents agree that the Probability and Statistics learning unit is difficult due to the misinterpretation of the problems, since there are different logics of resolution for the same problem, both for the students and teachers and for the authors of the exercises.

These applications were proposed in order to help the student and teacher in the process of raising and solving problems of the different themes of the already mentioned educational signature and campus.

After working with the applications previously shown, the following was found:

- It allowed the students to visualize the situations that were shown in the form of problems, helping them to improve in the approach of what they had to solve.
- There was an improvement in the time spent in solving the problems that were resolved.
- The responses obtained from the exercises that previously had to be done on the board, individually or in groups, could be confirmed.

It can be concluded that, when using these applications or web sites, the student's performance was higher and the error rate in terms of problem resolution dropped significantly. Equally important is that part of the students mentioned that it was easier for them to understand the issues and, in case of having an error, to notice the failure that was had.

### Questionnaire for evaluation of the use of applications

Table 4 shows the questions that were asked to the 32 students who worked for a month with the applications described above.

**Tabla 4.** Preguntas del cuestionario de evaluación para los estudiantes.

Después de haber trabajado con diferentes aplicaciones y páginas web en tu curso de Probabilidad y Estadística te pedimos responder las siguientes preguntas:

1. ¿En qué consideras que te ayudaron las aplicaciones?
2. ¿Recomendarías el uso de estas aplicaciones en el curso de Probabilidad?  
Sí / No ¿Por qué?
3. ¿Qué otra técnica propones para ayudar a la comprensión de los textos de los problemas de probabilidad?
4. Resuelve los problemas que aparecen en la hoja anexa titulada como “Examen de la unidad 3”. Puedes auxiliarte de una calculadora para hacerlo.

Fuente: Elaboración propia.

Regarding the evaluation questionnaire applied to the 32 students on the development of techniques for a better understanding of mathematical problems (specifically the subject of Probability and Statistics), it was found that 85% of students consider that the use of these tools reduces the time spent on solving problems in relation to when they only work with a notebook and a basic calculator. Seventy-six percent said it helped clarify what the problem calls for and 90% said that the applications allowed them to review how the events of the problems occur. In addition, 96% considered that if the exercises were related to the student's study area they would have a better understanding and interpretation of them. Eighty-four percent felt that a reduction in the number of exercises would have a positive impact on the understanding of the topics they address, because they mentioned that their teachers leave them a list of many problems and do not have enough time to analyze them. They are asking them about the problem, so they solve them using formulas, but without much reasoning.

Also, the entire sample of students recommended the use of applications because they can work with them on their cell phones anywhere. Seventy-six percent agree with the fact



that, if new technologies are used in mathematics teaching (particularly in the Probability and Statistics course), students would become more involved in the topics. Similar to the question of the use of new technologies, 90% found the development of recreational activities to solve problems of Probability and Statistics as an option for better learning.

For its part, 56% were interested in making readings related to mathematics that allow the development of critical thinking, this being another possible proposal different from the one presented in this article, which is directly related to the fact that 84% consider that a reduction in the number of exercises would have a positive impact on the understanding of the topics they address (because there is a common difficulty among students: the misinterpretation of the statements, some of which are confusing).

In relation to the way in which the work done in the classroom with the applications was evaluated, an exam was applied consisting of five problems, in which the students used the applications to solve it. This exam was worth ten points and is shown in table 4. The procedure used by the student, the strategies with which they worked and the interpretation of the results obtained were evaluated. The scale that was handled was from zero to 10, with passing grades being those corresponding to the interval of six to 10 and the remaining ones being rejected.

In Annex 1, the problems solved by the students are presented, which were taken from one of the textbooks that is indicated in the syllabus of Probability and Statistics. (Walpole, Myers y Ye, 2012; Meyer, 1999).

For the revision of the answers given by the students, a classification was made in four aspects. First, we wanted to know if they had distinguished the type of variable that was at stake in each problem, that is, if it was discrete or continued. Then the type of probability distribution to which these variables correspond was reviewed, according to their behavior. Then it was identified if the student had used any of the applications worked in class, for this he was asked to write on the questionnaire sheet the application (s) used. Finally, the points obtained in each problem were counted. Each had two paragraphs to solve, so each subsection was equivalent to a point. In total, the questionnaire was 10 points. Table 5 shows the concentration of the information expressed in the four aspects indicated.

**Tabla 5.** Clasificación de la información obtenida de las respuestas dadas por los estudiantes al cuestionario final.

Núm. de problema	Identificó la variable (discreta o continua)	Identificó la distribución de probabilidad	Usó alguna app o plataforma educativa	Operaciones correctas	Respuestas correctas
1	Sí (29/32)	Sí (28/32)	Sí (18/32)	Sí (29/32)	29/32
2	Sí (27/32)	Sí (27/32)	Sí (19/32)	Sí (27/32)	27/32
3	Sí (27/32)	Sí (27/32)	Sí (19/32)	Sí (27/32)	27/32
4	Sí (21/32)	Sí (24/32)	Sí (19/32)	Sí (19/32)	19/32
5	Sí (24/32)	Sí (23/32)	Sí (20/32)	Sí (20/32)	20/32

Fuente: Elaboración propia.

Twenty-seven of 32 students (84.4%) obtained a passing grade and one student did not show up for the exam.

The group average was 7.3 with a standard deviation of 2.6 and 68.7% of the scores were within one standard deviation with respect to the mean, which corresponds to what is indicated by the empirical rule (a standard deviation to the right and one on the left contains at least 68% of the data in the sample).

The results obtained with the group imply that the applications they used in the classes and outside of them were very helpful, since the students managed to make correct interpretations of the texts of the problems and were able to adequately solve the situations posed.

## Conclusions

The premise raised at the beginning of the investigation in relation to the difficulty that students have when learning Probability and Statistics lies in the poor understanding and interpretation of the problems raised in class. It was a corroborated premise, although other factors were found. It was found that there are several weak points that cause difficulties in learning the subjects of this subject, either because of the structure of the program of the subject (many subjects in a short time) or because of the type of teaching that predominates in the teachers they teach (very recostado in the conductismo), as well as in the form in which they are written up the problems that are extracted of the text books, which are formulated

by means of extensive and often elaborated texts. Another aspect is that most students do not consider a practical utility in the knowledge of the subject.

With the information collected, by surveys and questionnaires applied to students and the interview with teachers, it was decided to use a strategy that incorporated graphic material, use of websites and computer programs to analyze the contexts of the different problems that were worked in class. The work that was developed coincides with that pointed out by Sierpiska and Lerman (1996), regarding social constructivist learning.

In order for students to understand the different contexts that are addressed in the problems of probability, it is convenient to use videographic material such as that used in websites and applications, and thus the student has a more effective approach in the solution process.

On the other hand, the use of applications facilitated the understanding of the different problems that were solved, allowing the student to visualize the situations posed and not only work in the algebraic terrain and with the exclusive use of formulas.

It is concluded that, when working problems in different contexts, teachers should focus on helping students in the interpretations of texts, as well as to use different semiotic representations, such as graphic, algebraic, numerical and tabular representation.

The teacher must support the student so that he has different approaches in the solution process; It can start from the intuitive to reach the formal. It is also welcome to use different resources that allow the student to visualize the situations posed, as well as propose discussions that lead him to reflect on the elements that contain the problem and what is intended to be obtained. Therefore, it is essential to study the reasoning processes used by students, the skills that can be developed, and the procedures they can use.

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## Anexo 1

(Hoja que se entregó a los estudiantes para resolver los problemas de distribuciones de probabilidad).

**Lea con atención y use la herramienta y/o fórmula conveniente para resolver los siguientes problemas. Todo problema debe tener procedimiento de solución.**

1. Un escritor de libros comete, en promedio, 2 errores de procesamiento de texto por página en el primer borrador de su libro. ¿Cuál es la probabilidad de que en la siguiente página cometa:

a) ¿Cuatro o más errores?

b) ¿Ningún error?

2. Se estima que el número promedio de ratas de campo por acre, en un campo de 5 acres de trigo, es de 12. Calcule la probabilidad de que se encuentren menos de 7 ratas de campo.

a) en 1 acre dado;

b) en 2 de los siguientes 3 acres que se inspeccionen.

3. Suponga que la probabilidad de que una determinada persona crea un rumor acerca de las transgresiones de cierta actriz famosa es de 0.8. ¿Cuál es la probabilidad de que...

a) la sexta persona que escuche este rumor sea la cuarta en creerlo?

b) la tercera persona que escuche este rumor sea la primera en creerlo?

4.- Se estima que 4000 de los 10,000 residentes con derecho al voto de una ciudad están en contra de un nuevo impuesto sobre las ventas. Si se seleccionan al azar 15 votantes y se les pide su opinión,

a) ¿cuál es la probabilidad de que a lo sumo 7 estén a favor del nuevo impuesto?

b) ¿cuál es la probabilidad de que al menos 7 estén a favor del nuevo impuesto?



5. Un empleado va a su oficina en el centro de la ciudad. El tiempo promedio para un viaje solo de ida es de 24 minutos, con una desviación estándar de 3.8 minutos. Si se supone que la distribución de los tiempos de viaje está distribuida normalmente.

a) ¿Cuál es la probabilidad de que un viaje tome al menos 1/2 hora?

b) Calcule la duración mayor en la que se encuentra el 15% de los viajes más lentos.