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Artículos científicos

Actualización de contenidos en el campo disciplinar de matemáticas del componente propedéutico del bachillerato tecnológico: el caso de las funciones especiales

Updating of contents in the disciplinary field of mathematics of technological high school: the case of special functions

Atualização de conteúdos no campo disciplinar de matemática da componente preparatória do bacharelado tecnológico: o caso das funções especiais

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Resumen

El estudio de las funciones en el bachillerato en México se ha limitado únicamente a las elementales ignorando a las funciones hiperbólicas y especiales. Estas últimas, por lo general, se encuentran en términos de integrales, y para evaluarlas deben utilizarse algoritmos numéricos que se hallan fuera del alcance de los estudiantes de este nivel educativo. Por tanto, en este artículo se propone actualizar los contenidos en la asignatura propedéutica Matemáticas Aplicadas del campo disciplinar de matemáticas del bachillerato tecnológico en la que se agrega la función Lambert W al estudio de las funciones trascendentes. Además, se considera adicionar el estudio de las funciones hiperbólicas y especiales a través de aproximaciones matemáticas que se encuentran en términos de funciones elementales, lo que facilitará su implementación en *software* educativo como GeoGebra. Asimismo, se ofrecen cuatro ejemplos en donde se aplican las aproximaciones de las funciones que se consideran en la propuesta curricular, las cuales pueden servir de guía para mostrar aplicaciones relacionadas con la tecnología y el cotidiano de los alumnos.

Palabras clave: discurso matemático escolar, el cotidiano, graficado y aplicación de funciones matemáticas.

Abstract

The study of functions in high school has been limited only to elementary ones, ignoring hyperbolic and special functions. The latter are generally found in terms of integrals and numerical algorithms must be used to evaluate them, leaving them out of the reach of high school students. In this paper a content update is made in the course of applied mathematics of the high school in which the Lambert W function is added to the study of transcendent functions. In the same way, the curricular proposal considers the study of hyperbolic functions and special functions through mathematical approximations found in terms of elementary functions, which facilitates their implementation in educational software such as GeoGebra. In addition, in this work four examples are presented where the approximations of the functions that are considered in the curricular proposal the functions are applied and that can serve as a guide to show applications related to technology and the daily life of the students.

Keywords: school math speech, daily life, graphing matematical functions.

Resumo

O estudo das funções no ensino médio no México foi limitado apenas aos elementares, de modo que as funções hiperbólicas e especiais foram ignoradas. Estes últimos são geralmente encontrados em termos de integrais, e algoritmos numéricos que estão além do alcance dos alunos neste nível educacional devem ser usados para avaliá-los. Portanto, neste artigo propõe-se a atualização dos conteúdos da disciplina preparatória de Matemática Aplicada do campo disciplinar da matemática do bacharelado em tecnologia em que a função de Lambert W é agregada ao estudo das funções transcendentes. Além disso, considera-se agregar o estudo de funções hiperbólicas e especiais por meio de aproximações matemáticas encontradas em termos de funções elementares, o que facilitará sua implementação em softwares educacionais como o GeoGebra. Da mesma forma, são oferecidos quatro exemplos onde se aplicam as aproximações das funções consideradas na proposta curricular, que podem servir como um guia para mostrar aplicações relacionadas à tecnologia e ao cotidiano dos alunos.

Palavras-chave: discurso matemático escolar, cotidiano, representação gráfica e aplicação de funções matemáticas.

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Introduction

The use of functions becomes important in science and engineering, since the representation of any model, phenomenon or system is carried out through static equations (Baldor, 2011), Barnett (1994), differential equations (Edwards and Penney, 2011), Zill (1997) and integro-differentials (Wazwaz, 2011), while the behavior of these phenomena or systems can be expressed through algebraic and transcendent functions, which give rise to a graphical representation (Barnett, 1994). Without the existence of the functions, it would be impossible to carry out some type of analysis, such as stability analysis, frequency response (Ogata, 2003), among others.

For years - and before the 2004 reform in the technological baccalaureate system of the Industrial Technological Directorate (DGETI) in Mexico (DOF, 2004) the study of trigonometric functions (Barnett, 1994) began to be studied in the second semester (DOF, 2004, 2012) in the subject of geometry and trigonometry; in the third semester in analytical geometry (DOF, 2004, 2012; Kindle, 1994), and in the fourth semester in the subject of differential calculus (Ayres, Mendelson and Abellanas, 1994), (Casares, 2018); DOF, 2004, 2012). Likewise, integral calculus is taken in the fifth semester, where the different integration methods, the definite integral and their applications are presented, while in the sixth semester the subject of probability and statistics is studied (DOF, 2012). In that same semester, you can take the applied mathematics subject, which is also part of the disciplinary field of mathematics, preparatory and in turn optional (DOF, 2012). This chair is aimed at students who wish to continue their studies at the university level. In applied mathematics, mathematical modeling, calculus, and transcendent relationships are studied. The current study program of the Applied Mathematics subject has the deficiency of insufficiently addressing modeling using functions, since it is limited to classic examples, with a very fragmented vision of reality (DOF, 2012).

School math speech

School mathematical discourse (DME) is all the language that is introduced in a class, where students speak, expose their mathematical concepts, discuss ideas and solve problems (Soto, Gómez, Silva and Cordero, 2012). It is characterized by being hegemonic, utilitarian and devoid of frames of reference, thereby imposing meanings, arguments and procedures focused on mathematical objects (Soto et al., 2012; Soto and Cantoral 2014).

One way to visualize the DME is to identify everything that remains despite the innovations in relation to school mathematics because this innovation, deep down, does not modify what is being taught, but only how it is being taught. It is important to note that many mathematics textbooks used in the classroom are conceived under the DME (Uriza, Espinosa & Gasperini, 2015), which influences the teaching-learning processes.

The everyday

In a traditional way, the teaching of mathematics has been done in a decontextualized way, devoid of meaning and alien to the real world, since it has been ignored that its objective is to solve everyday problems and, where appropriate, science, hence that they should try to solve questions such as the following: what is this for me? Where am I going to use it?

In this sense, the daily life of the student, in general, we link it at school to personal situations, where there is some kind of exchange. For this reason, in textbooks (Olguín-Díaz and Sánchez-Linares, 2016) it is common to find typical exercises such as those of Rosita, Pepito and Juanita, who went to the corner store to buy 5 popsicles, a package of cookies and a chewing gum, which they paid with a $ 50 bill and are interested in knowing how much change they will give them.

However, this conception is limited because the use of functions should not be restricted exclusively to models and situations that are presented to the student in the classroom. Some examples of typical applications that are usually offered to the student are the growth of a population as a function of time or graphically representing the relationship of the cost of a product against its demand, among others. However, by presenting only these traditional problems in the classroom, DME is again encouraged because it appears that mathematics only fits the interpersonal realm.

On the other hand, the daily life that surrounds the student is very broad, since it not only includes monetary exchanges, but also physical, chemical and technological phenomena. Therefore, students could be interested in other phenomena, such as the forecast of the weather in the area where they live, the spread of a disease, innovation in the area of health sciences (Cordero, 2013; Vazquez-Leal et al., 2015b), the effects of temperature on construction materials (Treviño et al., 2004), the composition of cleaning products (Collado, Huallapacusi and Osores, 2005), the electromagnetic waves emitted and received by electronic devices (eg, ovens, cell phones, radio, television) (Proakis and Masoud, 2001; Haykin, 1994), the behavior of vibrations in a bridge with a metallic structure (Bermúdez, 2005), among others. Consequently, it is pertinent that the student has a broader panorama of the application of the functions to solve not only everyday situations, but also to understand the technology that exists around them.

Methodology

In this work, a documentary investigation was carried out in the study programs of the disciplinary field of mathematics in curricular structures of the technological baccalaureate in order to know the updates in the programmatic contents between agreements 345 and 653 in the databases of the Official Gazette of the Federation and on the website of the Sectorial Coordination of Academic Strengthening in Mexico (COSFAC).

The research included more than 60 books on pre-calculus, calculus and related areas of engineering (by different authors and publishers in the English and Spanish languages) to compare the published content. In some books (such as pre-calculation published by the same author) the updated and previous editions have been contrasted to verify the changes made. Among the books reviewed were manuals of mathematical functions, such as those by Abramowitz, Stegun and Romer (1988) and Oldham, Myland and Spanier (2010), in which the special functions were searched. In the references of this article, it has been considered to cite the best-known authors' books in which the content has been reviewed.

Likewise, databases such as Google Scholar, Scielo, Dialnet, Journal Citation Reports and Scimago were consulted. The search criteria were scientific articles on approximations and applications of special functions, with keywords such as special functions, approximative expressions for special functions and special functions applied.

Based on the above, a database was created in Excel where approximately 40 articles related to special functions, their applications, approximation of functions and asymptotic methods were recorded (since the mathematical approximations for these functions are based on these. and in general when an approximate solution is proposed for a differential equation), which were classified into a) special functions and b) applications in science and technology. From this classification, the feasibility to implement in high school and the ease to develop the functions in any educational software was considered.

Materials

For the presentation of the graphs and solved examples presented in this work, the mathematical software Maple 2015 was used. Likewise, a computer with Linux Ubuntu operating system (version 18.04.5 LTS) with Intel I7-7700 @ 3600GHz x 8 processor was used.

Results

Documentary research found that in the new editions of some textbooks (Baldor, 2011; Barnett, 1994; Edwards and Penney, 2011; Kindle, 1994; Leithold, 2012; Ogata, 2003; Olvera, 1991; Purcell, Rigdon and Varbeg, 2007; Stewart, 2015; Wazwaz, 2011; Zill, 1997), favorable changes are made in the pedagogy with which the thematic concepts are presented, with more demonstrative examples and with a greater number of exercises proposed. In other cases, the texts incorporate the use of mathematical software such as Maple (Fox, 2011), Matlab (Almenar, Isla, Gutiérrez and Luege, 2018), GNU Octave (Lie, 2019), GeoGebra (Mora-Sánchez, 2019), Excel (Torres-Remon, 2016), among others.

An analogous situation occurs in the study programs of the COSFAC when a content “revision” is made in subjects with a common core and specialty (DOF, 2004, 2012), since they generally focus more on the design of education strategies. learning, evaluation and didactic planning (Arceo, Rojas and González, 2010; Charur, 2016; Morales-Lizama, 2016), and the use of information and communication technologies (Álvarez and Mayo, 2009). However, in study programs of the disciplinary field of mathematics, in curricular structures of the technological baccalaureate of agreements 345 and 653, the introduction of new contents within the disciplinary field of mathematics was not proposed. In addition, it was found that the study of functions in high school has been limited only to algebraic and transcendent ones.

In reference to special functions, they were found to be very important in mathematical analyzes of science and engineering, since each of the different functions has different applications in different branches of science. For example, Fresnel functions (Oldham et al., 2010) have applications in optics. First and second class elliptical functions are applied in celestial mechanics (Fukushima and Kopeikin, 2014), electromagnetism (Brizard, 2009; Greenhill, 1907), and planetary orbits (Fukushima and Kopeikin, 2014). The error function is found applied in transport phenomena (Bird, Stewart and Lightfoot, 2002) and digital communications (Sadiku, 1998), while the probability density function in statistics is found in engineering areas (González and Woods , 1996), social sciences (Hernández-Sampieri, Fernández-Collado and Baptista, 2010), health sciences (Wayne, 2000), among others.

Table 1 summarizes some of the special functions used in science and engineering. Their names, symbols, some of their applications, their classification and the references where they are are presented. These functions have a high potential to be included in the applied mathematics subject of the technological baccalaureate, since they have many applications related to the student's daily life.

In the same way, hyperbolic functions have been added to this table. In the research carried out, we found that the textbooks of the upper secondary level do not take them into account, since they focus their attention on the derivation of algebraic and transcendent functions. However, the derivation rules for these functions are practically the same.

Tabla 1. Funciones especiales y sus aplicaciones

Nombre de la función	Símbolo	Aplicaciones	Clasificación	Referencia
Lambert W	W(x)	Electrónica, química	Trascendente	Corless *et al*. (1996), Johansson (2020)
Función signo	sgn(x)	Análisis de funciones	Especial	Abramowitz *et al*. (1988), Sandoval-Hernández *et al*. (2018)
Integrales de Fresnel	S(x), C(x)	Óptica, electromagnetismo	Especial	Hecht y Zajac (1987), Balassone y Romero (2015)
Función error	erf(x)	Comunicaciones, fenómenos de transporte	Especial	Bird *et al*., (2002), Proakis y Masoud (2001), Haykin (1994)
Función de distribución acumulativa de probabilidad	p(x)	Estadística	Especial	González y Woods (1996), Hernández-Sampieri *et al*. (2010) Wayne (2000)
Funciones elípticas	F(x), E(x)	Mecánica, teoría electromagnética	Especial	Fukushima y Kopeikin (2014)
Función Gamma	$\Gamma(x)$	Matemáticas, estadística	Especial	Abramowitz *et al*. (1988)
Funciones hiperbólicas	sinh, cosh tanh, csch sech, tanh	Mecánica, electricidad, matemáticas	Combinación de funciones exponenciales	Barnett (1994), Leithold (2012), Stewart (2015)

Fuente: Elaboración propia

Regarding the Lambert W function, we find that currently the trigonometry and precalculus books do not include its study and do not even mention its existence. Currently, this function is widely used in solving engineering problems, which is why it has been published in various scientific articles. In Corless et al. (1996) and Vazquez-Leal et al. (2020) a brief historical review of this function is obtained.

Table 2 offers the catalog of functions proposed for the applied mathematics course, where the type of function to be used, either exact or approximate, is presented, as well as the reference from which they were obtained.

Tabla 2. Funciones especiales y sus aproximaciones

Nombre de la función	Tipo de expresión	Publicado en
Lambert W	Aproximación	Vázquez-Leal *et al*. (2019)
Función signo	Función exacta	Abramowitz *et al*. (1988)
Integrales de Fresnel	Aproximación	Sandoval-Hernández *et al*. (2018)
Función error	Aproximación	Sandoval-Hernández *et al*. (2019a)
Función de densidad de probabilidad acumulativa	Aproximación	Sandoval-Hernández *et al*. (2019a)
Funciones elípticas de primera y segunda clase	Aproximación, Metodología*	Vázquez-Leal y Sarmiento-Reyes (2015b) Sandoval-Hernández *et al*. (2019b)
Función Gamma	Aproximación, Metodología*	Zhen-Han y Jing-Feng (2018) Komla-Amenyou (2018) Vázquez-Leal y Sarmiento-Reyes (2015b) Vázquez-Leal *et al*. (2019) Sandoval-Hernández *et al* (2019a)
Funciones hiperbólicas	Función exacta	Barnett (1994)
*Nota: Aproximaciones obtenidas con las metodologías matemáticas aproximativas publicadas según la referencia.		

Fuente: Elaboración propia

The approaches presented in the references contained in Table 2 have been selected because they are in terms of elementary functions, which makes it easy to implement them in any educational software, making it possible to study them in the classroom. In the literature there are more approximations for the functions in table 2, although some of them are found in terms of other special and unknown functions that make their implementation difficult in the technological high school using specialized mathematical software.

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The content update proposal

In Figure 1 we propose the new content suggested for the applied mathematics subject so that the student reinforces the concepts related to the management of functions and the modeling of environmental situations that contribute to developing the disciplinary competences that are part of the profile of the graduate of the EMS (DOF, 2008).

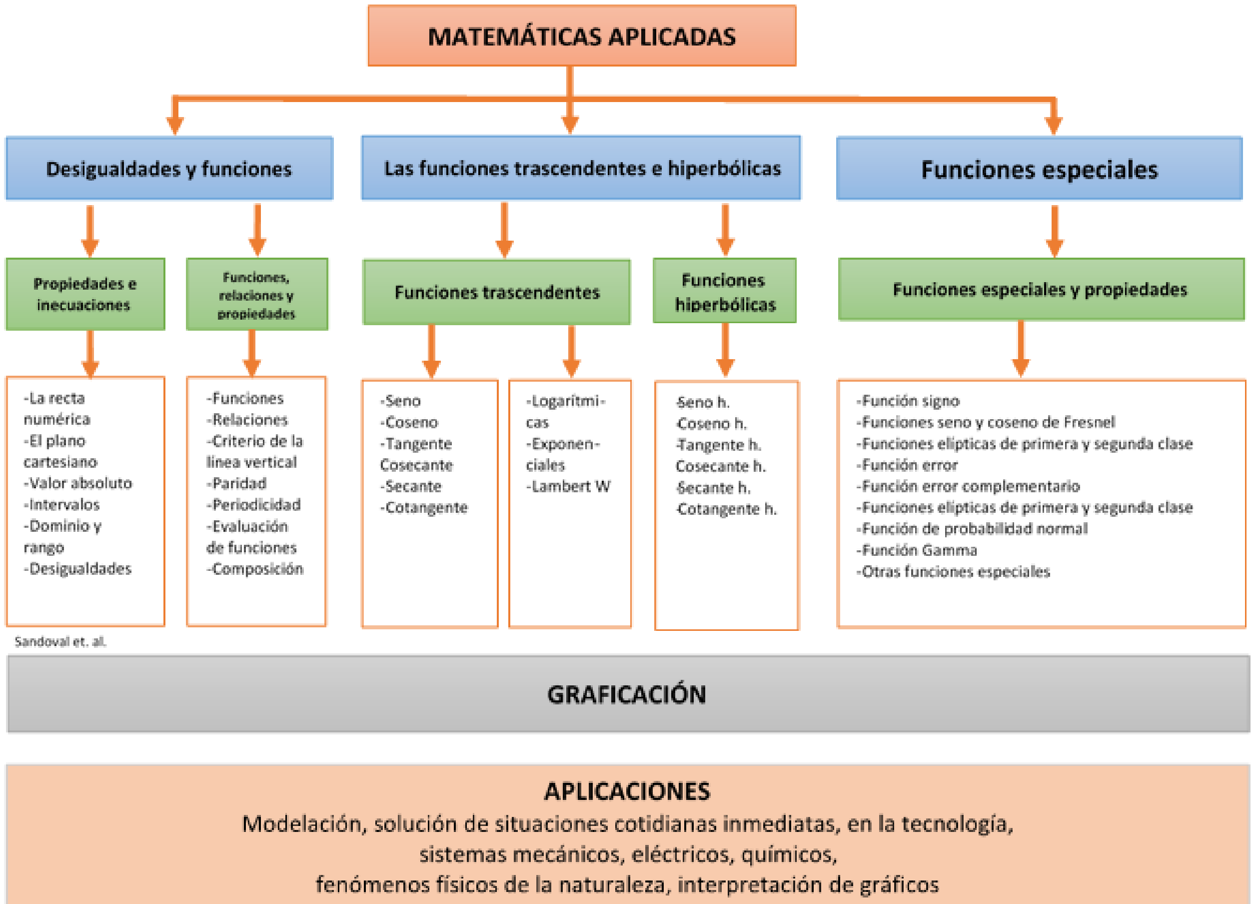
In the first block, inequalities and functions are studied, where knowledge of the number line and the Cartesian plane are recovered. In addition, the absolute value, intervals, domain and range of a function and inequalities are presented. In the study of functions, it is necessary to identify the domain and counter-domain of a function, as well as how to interpret and represent them algebraically and graphically. In addition, the criterion of the vertical line is studied to identify whether a graph represents a function or a relationship. The periodicity and parity of a function are also studied.

In the second block, the transcendent and hyperbolic functions are studied, as well as the characteristics of the circular, hyperbolic, logarithmic and exponential trigonometric functions. The Lambert W function has been added to the group of transcendent functions.

The third block presents the group of special functions that includes the sign function, Fresnel functions, error and complementary function, elliptic functions, normal probability function and Gamma function. In addition, the possibility of including more special functions opens.

In the proposal that we present in this article, the graphing of the functions throughout the semester is considered at all times. It is intended that the student identify and interpret the different properties of functions such as parity, periodicity of a function, as well as the characteristics of each of the transcendent functions. Finally, emphasis is placed on presenting and proposing to the students different case studies, which include some type of modeling using some of the different transcendent, hyperbolic, special and algebraic functions.

Figura 1. Programa de estudios propuesto para la asignatura de matemáticas aplicadas



Fuente: Elaboración propia

Discussion

In general, at all educational levels (including the technological baccalaureate), the use of the textbook is common, which has a wide variety and is disseminated by prestigious publishers. This work has served as a guide and source of additional information, and is a means by which educational consensus is built. "It serves to introduce an ideology and to legitimize specific content and forms of school knowledge" (Uriza et al., 2015). However, in the area of educational mathematics, different proposals have been made to facilitate the understanding of the management of functions. For example, to understand the behavior of a function, Cordero Osorio (2002) analyzes that the linearity situation of the polynomial has the intention of relating the tangent line with the behavior of the function. In the case of the equation

$$y = ax^2 + bx + c, \qquad (1)$$

Students must construct a meaning for the linear part of the polynomial, that is, for the linear and independent terms in relation to the trend behavior of the graph of the polynomial (quadratic term). The reconstruction of meaning consists of two aspects: identifying the property of linearity and establishing it as an argument (Uriza, 2000). In this way, the linearity of the polynomial consists in that the linear part of any polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0, (2)$$

is the tangent line to the polynomial that passes through the point (0, P (0)) (Cordero Osorio, 2002).

There are other interesting proposals for learning mathematics. In Cordero Osorio and Domínguez García (2001) a design of the situation of sinusoidal asymptotes was published, taking into account the students' previous experiences. In the first place, graphic knowledge: "Determine if the graphs presented below have asymptotic behavior." Second, algebraic experiences: "Relate the following functions with their respective graphs" and, third, analytics: "Provide a definition of the asymptote of a function. In Cordero and Suárez (2005) a didactic design is offered that exemplified graphic modeling, where the parable and graphic models related to situations of the displacement of a person were resignified.

In the works of Uriza (2000), Cordero Osorio (2009) and Cordero Osorio and Domínguez García (2001) the student is provided with elements that enrich the way of understanding the variational behavior of functions. Likewise, the student learns to recognize the behavior of the function, regardless of the type being analyzed, whether it is continuous, discontinuous, polynomial or transcendent. However, in all these works the study of elementary functions is only addressed algebraically and geometrically, since the analyzes carried out focus on these two characteristics. We consider that these didactic proposals that different authors have proposed should be accompanied by physical everyday situations that are meaningful for students (Filobello-Nino *et al*., 2017a; Filobello-Nino *et al*., 2017b; Serway y Jewett, 2018; Vázquez-Leal *et al*., 2017).

It is important to emphasize that the textbooks of the technological high school tend to do this type of analysis (algebraic and geometric), although they do not manage to completely overcome the SMD because for a long time the physical meaning has been ignored in everyday life. Consequently, the application of mathematics at the high school level to everyday life (Soto et al., 2012; Soto and Cantoral, 2014) can be considerably expanded.

The special functions and the group of approximate functions

In upper secondary education and in technological baccalaureate, in previous years it has not been possible to include the behavior of special functions in the study programs because it is not easy to evaluate or graph them simply with pencil and paper or with educational software without having a algebraic expression. For this purpose we include in this section the set of approximations for the special functions that have been proposed in the applied mathematics course.

The notation for these approximations have the tilde symbol (~), although for practical purposes they are considered functions. This section presents its approximation and its graph. For more details you can consult the references of this article.

The sign function

The sign function is defined as

$$\mathrm{sgn}(x) = \begin{cases} 1, \mathrm{si} \ > 0, \\ 0, \mathrm{si}\ x = 0, \\ -1, \mathrm{si}\ x < 0, \end{cases} \qquad (3)$$

The Lambert W function

The Lambert function W is defined as the inverse of the transcendental function

$$y(x)\mathrm{e}^{y(x)} = x. \qquad (4)$$

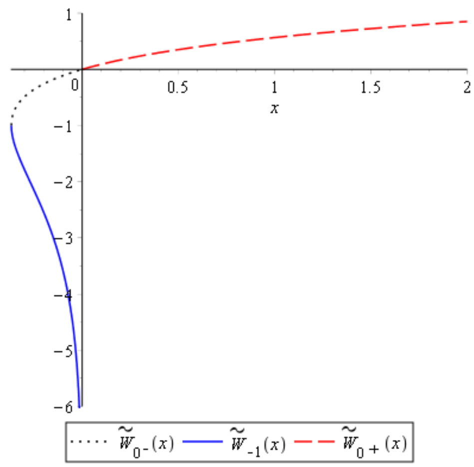
Solving for we have

$$y(x) = W(x), \qquad (5)$$

where x can be a real or complex number and $W(x)$ It is the Lambert W function. This function belongs to the group of transcendent functions and it is multi-valued because in the domain of real numbers it has two branches. In the figure you can see the branches $W_0(x)$ and $W_{-1}(x)$.

$W_0(x)$ is the main branch that satisfies the condition $W(x) \geq -1$. This branch can be divided into two zones $W_{0^-}(x)$ and $W_{0^+}(x)$, because in this place the function is multivalued. $W_{-1}(x)$ is the lower branch and satisfies the condition $W(x) \leq -1$. The two branches of the function are in (-1/*e*,-1). This way in the interval $-1/e < x < 0$ there are two possible values of $W(x)$. The first to $W_{0^-}(x)$ and the second for $W_{-1}(x)$. In figure 2 the two branches of the Lambert W function are presented.

Figura 2. Función Lambert W



Fuente: Elaboración propia

Vázquez-Leal *et al*. (2019) the approximation for Lambert W was proposed for both branches. The subscript zero refers to the top branch. See that the top branch has four functions for the intervals shown:

$$\widetilde{W_0}(x) = \begin{cases} \widetilde{W_{01}}(x), -\mathrm{e}^{-1} \leq x < 1, \\ \widetilde{W_{02}}(x), 1 \leq x < 40, \\ \widetilde{W_{03}}(x), 40 \leq x \leq 2000, \\ \widetilde{W_{04}}(x), 2000 \leq x. \end{cases} \tag{6}$$

The subscript -1 refers to the lower branch, which is made up of 3 approximations:

$$\widetilde{W_{-1}}(x) = \begin{cases} \widetilde{W_{-11}}(x), -\mathrm{e}^{-1} \leq x < -0.34, \\ \widetilde{W_{-12}}(x), -0.34 \leq x < -0.1, \\ \widetilde{W_{-13}}(x), -0.1 \leq x < -0.0001, \end{cases} \tag{7}$$

Due to the number of algebraic terms, the number of approximations and the space available, the appendix presents each of the approximations for the two branches of the Lambert W function. If you want to increase the accuracy of (6) and (7), you can use Fritz's iterative algorithm

$$z_n = \ln\left(\frac{x}{W_n}\right) - W_n,$$

$$e_n = \left(\frac{z_n}{1 + W_m}\right)\left(\frac{2(1 + W_n)(1 + W_n + \left(\frac{2}{3}\right) - z_n}{2(1 + W_n)(1 + W_n + \left(\frac{2}{3}\right) - z_n}\right),$$

$$W_{n+1} = W_n(1 + e_n), \qquad (8)$$

where n is the number of iterations e_n. In Corless et al. (1996) and Vázquez-Leal et al. (2019) details of the use of this algorithm are presented.

Comprehensive Fresnel functions

The approximations that are proposed for the two Fresnel integrals that allow evaluating them are expressed by

$$\tilde{S}(x) = (-\frac{\cos\left(\frac{\pi|x|^2}{2}\right)}{\pi\left(|x| + 16.73127745\pi \mathrm{e}^{-1.5763886076\pi\sqrt{|x|}}\right)} + \frac{8}{25}\left(1 - \mathrm{e}^{-0.6087077494307\pi|x|^3}\right) + \frac{2}{25}\left(1 - \mathrm{e}^{-1.714028381654\pi|x|^2}\right) + \frac{1}{10}\left(1 - \mathrm{e}^{-0.9\pi|x|}\right)\mathrm{sgn}(x), \qquad (9)$$

$$\tilde{C}(x) = (-\frac{\sin\left(\frac{\pi|x|^2}{2}\right)}{\pi\left(|x| + 20\pi \mathrm{e}^{-200\pi\sqrt{|x|}}\right)} + \frac{8}{25}\left(1 - \mathrm{e}^{-0.69\pi|x|^3}\right) + \frac{2}{25}\left(1 - \mathrm{e}^{-4.5\pi|x|^2}\right) + \frac{1}{10}\left(1 - \mathrm{e}^{-1.552940682\pi|x|}\right)\mathrm{sgn}(x), \qquad (10)$$

for $-\infty < x < \infty$.

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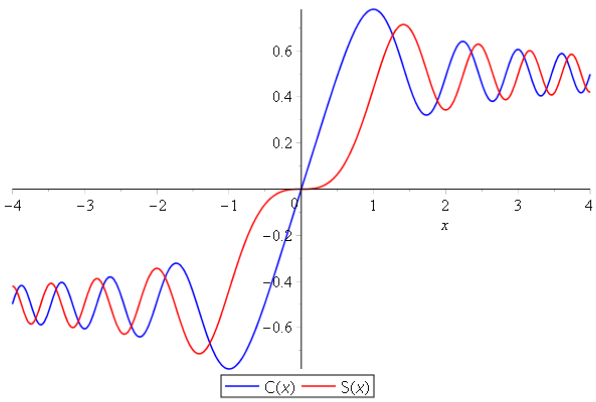
Fresnel sine and cosine integrals are shown in Figure 3.

Figura 3. Integrales de Fresnel



Fuente: Elaboración propia

Error function

For the error function we will use the expression

$$\widetilde{\operatorname{erf}}(x)=\frac{2}{1+e^{\xi(x)}}-1-\infty<x<\infty, \tag{11}$$

$$\xi(x)=\frac{-1}{5670}\left(\frac{105\pi^4-9328\pi^3+116928\pi^2-483840\pi+645120}{\sqrt{\pi^9}}\right)x^9$$

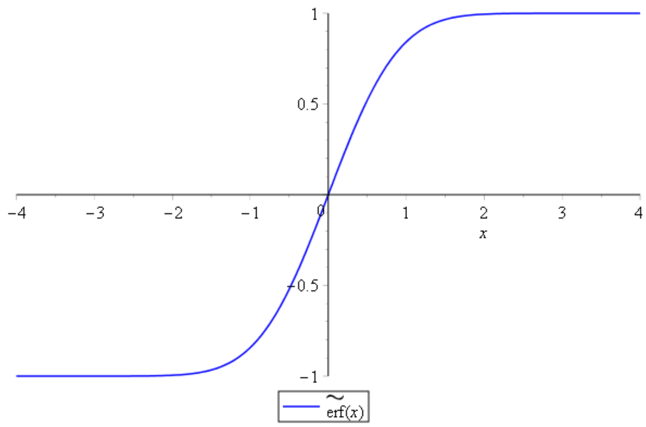
$$+\frac{2}{315}\left(\frac{15\pi^3-532\pi^2+3360\pi-5760}{\sqrt{\pi^7}}\right)x^7$$

$$-\frac{2}{15}\left(\frac{3\pi^2-40\pi+96}{\sqrt{\pi^5}}\right)x^5+\frac{4}{3}\left(\frac{\pi-4}{\sqrt{\pi^3}}\right)x^3-\frac{4}{\sqrt{\pi}}x.$$

The complementary function of this function is defined as the complementary error function erfc (x), given by erfc (x) = 1-erf (x).

In figure 4 we can see that erf (x) is an odd function.

Figura 4. Función error



Fuente: Elaboración propia

Normal probability distribution function

To obtain the numerical value of the probability without using the classic tables we will use the approximation

$$\tilde{P}(x) = \frac{1}{1 + e^{\zeta(x)}}, -\infty < x < \infty,$$

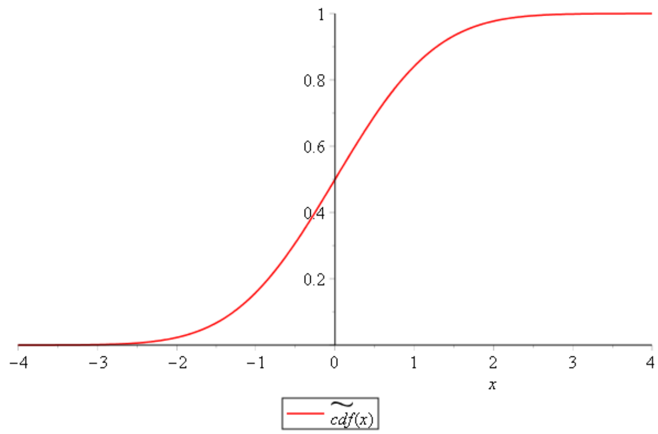
$$\zeta(x) = \frac{-\sqrt{2}}{181440}\left(\frac{105\pi^4 - 9328\pi^3 + 116928\pi^2 - 483840\pi + 645120}{\sqrt{\pi^9}}\right)x^9$$

$$+\frac{\sqrt{2}}{2520}\left(\frac{15\pi^3 - 532\pi^2 + 3360\pi - 5760}{\sqrt{\pi^7}}\right)x^7$$

$$\frac{-\sqrt{2}}{60}\left(\frac{3\pi^2 - 40\pi + 96}{\sqrt{\pi^5}}\right)x^5 + \frac{\sqrt{2}}{3}\left(\frac{\pi - 4}{\sqrt{\pi^3}}\right)x^3 - \frac{2\sqrt{2}}{\sqrt{\pi}}x. \tag{12}$$

Its graph is shown in figure 5.

Figura 5. Función de distribución acumulativa de probabilidad



Fuente: Elaboración propia

First and Second Class Elliptical Complete Integral Functions

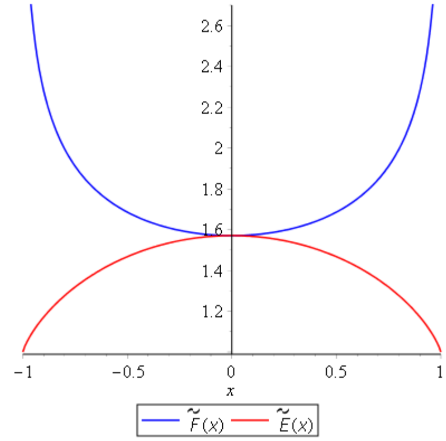
To evaluate the first and second class complete elliptical integrals we present:

$$\tilde{F}(x) = \frac{11}{7} + \ln\left(\frac{\sqrt[23]{\left(1-\frac{31}{47}x^2\right)^2}\ \sqrt[46]{\left(1-\frac{7}{38}x^2\right)^9}}{\sqrt[35]{(1-x^2)^{17}}}\right), \tag{13}$$

$$\tilde{E}(x) = \frac{360}{779}\sqrt{1-\frac{280}{339}x^2} + \frac{61}{118}\sqrt{1-\frac{245}{694}x^2} + \frac{159}{751}\sqrt{1-\frac{157}{157}x^2} + \frac{19}{50}\sqrt{1-\frac{123}{4318}x^2}. \tag{14}$$

Figure 6 shows the graphs of the first and second class elliptic functions.

Figura 6. Funciones elípticas de primera y segunda clase



Fuente: Elaboración propia

Gamma function

To evaluate $\Gamma(x)$ the approximations given by

$$\tilde{\Gamma}(x) = \begin{cases} \tilde{\Gamma}_1(x), x < 0 \\ \tilde{\Gamma}_2(x), 0 < x \leq 2.043 \\ \tilde{\Gamma}_3(x), x > 2.043 \end{cases} \quad (15)$$

with

$$\tilde{\Gamma}_1(x) = \frac{\sqrt{2\pi}\left(\frac{-\mathrm{e}}{x}\right)^{-x}}{2\sqrt{-x}\sin(\pi x)}(1 - 0.340198059\mathrm{e}^{7.511328218x} -$$

$$(0.259363253\mathrm{e}^{1.320358968x}), x < 0,$$

(16)

$$\tilde{\Gamma}_2(x) = \frac{1}{\mathrm{e}^{\varphi(x)} - 1}, 0 < x \leq 2.043,$$

$$\varphi(x) = 0.2084721215x\mathrm{e}^{-0.6941829682x^2} +$$

$$0.6902220099x\mathrm{e}^{-0.1818111523x^2} + 0.08383862837\mathrm{e}^{-1.832856342x^2}.$$

(17)

$$\tilde{\Gamma}_3(x) = \sqrt{2\pi(x-1)}\left(\frac{x-1}{\mathrm{e}}\right)^{x-1}\left(1 + \frac{1}{r(x)}\right)^{\left(\frac{1}{12}\right)}\mathrm{e}^{\left(-\frac{7}{\varepsilon(x)} - \frac{1531}{\zeta(x)}\right)}, x > 2.043,$$

$$\varepsilon(x) = 720(x-1)^3 - \frac{900}{49}(x-1),$$

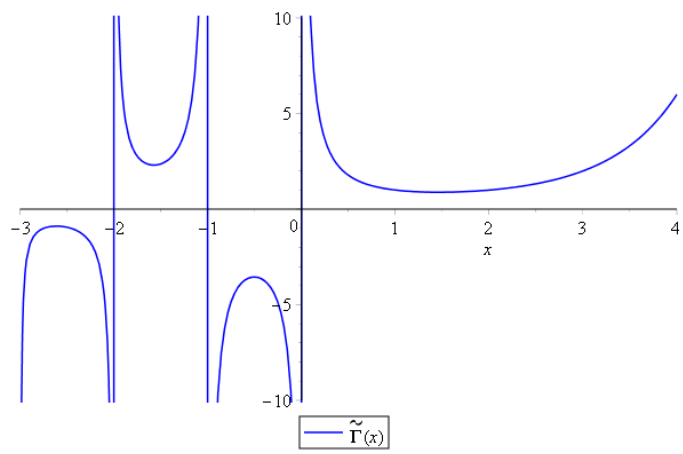
$$\zeta(x) = 1975680(x-1)^7 + \frac{34595736000}{16841}(x-1)^5 - \frac{10219256619062120}{3687050653}(x-1)^3 + \frac{1556259293563438478000}{186280860141519}(x-1),$$

$$r(x) = x - \frac{3}{2} + \big(7.664112723(x-1)\big)\mathrm{e}^{-7.787686838(x-1)} + +\big(0.0223051314(x-1)\big)\mathrm{e}^{-4.708891117(x-1)}.$$

(18)

Figure 7 shows the graph $\Gamma(x)$ for $-\infty < x < \infty$ in the domain of real numbers.

Figura 7. Función $\Gamma(x)$ aproximada



Fuente: Elaboración propia

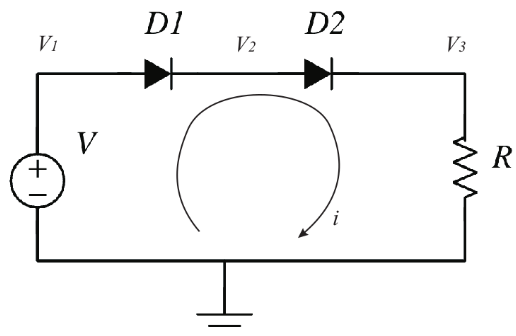
Study cases

Example 1. Determine the loop current in a circuit with two diodes in series

In analog electronic circuits courses, rectifier diode polarization is studied using a circuit with a diode, a resistor, and a voltage source (Boylestad & Nashelsky, 2009). In a traditional way, the diode bias point is determined where the load line of the circuit and the rectifier diode curve intersect; however, it is possible to determine this solution point using a numerical algorithm such as Newton Raphson (Burden and Faires, 1996).

In Sandoval-Hernández et al. (2019b) set out to find the solution of this circuit using two rectifier diodes in series with a single independent voltage source and a resistor with numerical values for each element of $V = 3V, R = 5\Omega, I_{s1} = 1E-12, I_{s2} = 1E-9, V_T = 25.86\text{m}V$, using the classical perturbation method (figure 8). The solution for the polarization point obtained in Sandoval-Hernández et al. (2019b) was algebraic, in terms of the circuit parameters. In this article we will proceed to show a different alternative solution for this circuit in order to overcome the DME using the Lambert W function.

Figura 8. Circuitos con dos diodos rectificadores en serie



Fuente: Sandoval-Hernández *et al*. (2019b)

Solution: Diodes are assumed to have different saturation currents. Using the Kirchoff stress law (LVK) Alexander and Sadiku (2013), the following is obtained:

$$Ri + 2V_T\ln(i) - V_T\ln(I_{s1}) - V_T\ln(I_{s2}) - V = 0, \qquad (19)$$

as V_T is thermal voltage, I_{s1}, I_{S2} are the saturation currents in each of the diodes.

Equation (19) is nonlinear and cannot be solved using conventional algebraic methods. We will use the Lambert W function to solve.

$$Ri + 2V_T\ln(i)V_T\ln(I_{s1}I_{s2}) + V.(20)$$

To simplify the algebraic calculations we do $K = V_T\ln(I_{s1}I_{s2}) + V$ in (20) to get

$$Ri + 2V_T\ln(i) = K. \qquad (21)$$

Solving algebraically, we have

$$\frac{-2V_T \ln(i) + K}{Ri} = 1. \qquad (22)$$

Multiplying (22) by $e^{\frac{K}{2V_T}}$,

$$\left(\frac{-2V_T \ln(i) + K}{Ri}\right) e^{\frac{K}{2V_T}} = e^{\frac{K}{2V_T}}. \qquad (23)$$

Manipulating the terms algebraically

$$\left(-\ln(i) + \frac{K}{2V_T}\right) e^{-\ln(i) + \frac{K}{2V_T}} = \frac{R}{2V_T} e^{\frac{K}{2V_T}}. \qquad (24)$$

Applying the Lambert function W and solving for i and substituting the value of K, we have the algebraic expression as a function of all the parameters of the circuit to find analytically the current of the circuit given by

$$i = e^{\frac{-1}{2V_T}\left[2V_T W\left(\frac{R}{2V_T} e^{\frac{V_T \ln(I_{S1} I_{S2}) + V}{2V_T}}\right) - V_T \ln(I_{S1} I_{S2}) - V\right]}. \qquad (25)$$

Using (6) and substituting the values given for each element of the circuit in (25) we obtain the value of $i = 0.3604662014$ A.

Example 2. Factorial of a number with a decimal or negative point

In the textbooks for statistics (Alanís-Martínez, 2016; Espinoza-Casares, 2017) of the technological high school we have that the positive integer factorial n is represented by $n!$. However, nothing is said about the factorial for negative or decimal point numbers. The function $\Gamma(x)$ helps us solve this problem. We will find the factorial of -3.79 using (15). This is, $(-3.79)! = 0.2975244944$.

Example 3. Mutual inductance of two parallel coils

In Nalty (2011) the mutual inductance of two separate coils along the axis was calculated z, at a distance $\frac{l}{2} = 0.5m$ of origin, with radius $L = R = 0.5\, m$, with magnetic allowance $\mu = 4\pi E - 7NA^{-2}$, with mutual inductance of $M = 7.093E - 8H$. Figure 9 shows the mutual inductance diagram of two coaxial coils.

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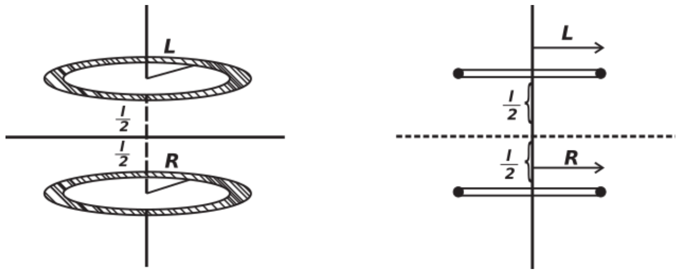
Figura 9. Inductancia mutua de 2 bobinas paralelas



Fuente: Elaboración propia

The equation that allows to find the mutual inductance of this configuration is given by

$$M = 2\mu\frac{a+b}{b}\left(\left(1-\frac{\beta^2}{2}\right)F(x)-E(x)\right), (26)$$

with

$$a = \frac{L^2+R^2+l^2}{(LR)^2}, b = \frac{2}{LR}, x = \sqrt{\frac{2b}{a+b}}. (27)$$

Substituting the numerical values in (26) and (27) and using $\tilde{F}(x)$ y $\tilde{E}(x)$, the mutual inductance $M = 7.092E-8H$.

Example 4. Determination of the probability in a population

In Wayne (2000) the probability of this case study was calculated using the classic tables for the normal distribution. We will use (16) to determine it.

The weight of a population sample has approximately a normal distribution with a mean of 140 pounds and a standard deviation of 25 pounds. What is the probability of randomly selecting a person weighing in the range of 100 and 170 pounds?

Solution: Let the transformation to typify

$$z = \frac{X-\mu}{\sigma}, (28)$$

As X is the random variable of the population weight, μ is the average, σ the standard deviation. Substituting $X = 100{,}170$ lb, $\mu = 140$lb, $\sigma = 25$lb en 38, you have $z_1 = -1.6$, $z_2 = 1.2$. Substituting in (12), we have $\widetilde{P_1}(-1.6) = 0.0548628902$, $\widetilde{P_2}(1.2) = 0.884768632$, respectively. Consequently, the probability $\tilde{P}(-1.6 \le z \le 1.2) = 0.830013973$.

Limitations

To implement this curricular proposal, it is necessary that the teachers who teach mathematics subjects have the academic profile related to the disciplinary field of mathematics, according to the selection of teachers by the System Unit for the Career of Teachers and Teachers, that is In other words, they must have the necessary university and teaching academic training that allows them to design and apply other didactic strategies in addition to those that currently exist in the teaching of functions. In this way, the facilitators will be able to recover from their respective professional fields (chemistry, physics, as well as mechanical, electrical, civil engineering, etc.) the application of these functions and propose them in the classroom to the students in order to promote learning significant (Arceo *et al*., 2010; Charur, 2016; Morales-Lizama, 2016).

Conclusions

The objective of this work was to present a curricular proposal for the preparatory subject of applied mathematics of the technological baccalaureate in order to reinforce the teaching of functions and their characteristics. In this sense, concepts such as domain, range, absolute value, periodicity and parity, among others, have been given great importance, since their understanding is necessary to study the behavior of mathematical functions.

Now, the addition of the Lambert W function to transcendent functions is proposed, since it belongs to this group of functions and, to date, it has not been given its place in textbooks. This function has algebraic properties that help to perform other algebraic clearances that are not presented in the traditional algebra and precalculus books. Likewise, it is proposed for the first time to add in the study plans the incorporation of hyperbolic and special functions, which have many applications in engineering found in the student's daily life, such as in the fundamentals of household electrical appliances, the phenomena physical, constructions, etc.

For the study of special functions, a series of approximations have been presented that are found in terms of elementary expressions, making it possible to study them in the classroom and avoiding the possible use of mathematical software that incorporates built-in routines that would make it look like a black box.

However, to implement this curricular proposal it is necessary that teachers in the area of mathematics have a good academic and pedagogical training, and are enthusiastic to investigate practical examples of applications in the student's daily life. In this way, each facilitator will be able to take advantage of the necessary didactic strategies using their teaching skills to make the knowledge that the students will acquire meaningful.

We consider that the study of special and hyperbolic functions will enrich the perspective of the technological baccalaureate student regarding the everyday life that surrounds him because the use of functions is not limited exclusively to arithmetic and algebraic problems that have been presented in a traditional way within the classroom. Therefore, it is expected that this proposal will encourage students to study a degree related to the exact sciences and engineering, and that in due course they will contribute to the development and technological progress of our country.

Future Research Lines

Pilot didactic strategies must be designed that allow the implementation of the didactic proposal presented in this work in order to carry it out, without losing sight of the previous knowledge that students have on the management and graphing of elementary functions. In addition, it is important to carry out a statistical analysis of the learning expected in the pilot test to make any possible adjustments. For this, it will also be necessary to design the evaluation instruments to collect the information that we wish to obtain. Likewise, it is important to suggest to the different authors who publish high school textbooks that they include the functions proposed in this work in order to promote their teaching at this educational level.

This curricular proposal, in short, can be extended to the first semesters of university education, where pre-calculus concepts seen in high school are retaken. Many students who enter university institutions and who pass the first semester continue to be unaware of the existence of special functions.

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Apéndice

```
%%Implementación de la función Lambert W.
%%%%%%%%%%%%%%%%%%%%%%%%%%%% Rama baja
%%%% Primer intervalo, -exp(-1) <= x and x < -.34
```

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$\widetilde{W_{11}}(x) =$ (-7.874564067684664 + (-63.11879948166995 + (-168.6110850408981 - 150.1089086912451 * x) * x) * x) * x * (x + 0.3678794411714423) / (1 + (15.9767983949761 + (98.26612857148953 + (293.9558944644677 + (430.4471947824411 + 247.8576700279611 * x) * x) * x) * x) * x) - 1;

%%%% Segundo intervalo, -.34 <= x and x < -.1
$\widetilde{W_{12}}(x) =$ (-1362.78381643109 + (-1386.04132570149 + (11892.1649836015 + 16904.0507511421 * x) * x) * x) * x / (1 + (251.440197724561 + (-1264.99554712435 + (-5687.63429510978 - 2639.24130979 * x) * x) * x) * x);

%%%% Tercer intervalo , -.1 <= x and x < -0.1e-4
$\widetilde{W_{13}}(x) =$(1.01999365162218 + (-12.6917365519443 - 45.1506015092455 * x) * x) * x / (1 + (-22.9809693297808 + (-104.692066099727 - 95.2085341727207 * x) * x) * x) + log(-x) - log(-log(-x)) + log(-log(-x)) / log(-x);

rbaja_W1= piecewise(-exp(-1) <= x and x < -.34, $\widetilde{W_{11}}(x)$, -.34 <= x and x < -.1, $\widetilde{W_{12}}(x)$, -.1 <= x and x < -0.1e-4, $\widetilde{W_{13}}(x)$);

%% Rama alta %%

%%%% Primer intervalo, -exp(-1) <= x and x < 1
$\widetilde{W_{01}}(x) =$ -(-(2/3) + 0.70710678118655 * (1 + 2.718281828459 * x) ^ (-0.5) - 0.05892556509888 * sqrt(1 + 2.718281828459 * x) + (x + 0.36787944117144) * (0.050248489761611 + (0.11138904851051 + 0.040744556245195 * x) * x) / (1 + (2.7090878606183 + (1.551092259782 + 0.095477712183841 * x) * x) * x)) / ((1/3) + 0.70710678118655* (1 + 2.718281828459* x) ^ (-0.5) - 0.05892556509888 * sqrt(1 + 2.7182818284591 * x) + (x + 0.36787944117144) * (0.050248489761611 + (0.11138904851051 + 0.040744556245195 * x) * x) / (1 + (2.7090878606183 + (1.551092259782+ 0.095477712183841 * x) * x) * x));

%%%% Segundo intervalo, 1 <= x and x < 40
$\widetilde{W_{02}}(x)$ = 0.160004963651493 * log(1 + (5.950065500550155 + (13.96586471370701 + (10.52192021050505 + (3.06529425426587+ 0.1204576877 * x) * x) * x) * x) * x);

%%%% Tercer intervalo, ,40 <= x and x < 20000
$\widetilde{W_{03}}(x)$= 0.09898045358731312 * log(1 + (-0.3168666425e12 + (0.3420439800e11 + (-0.1501433652e10 + (0.3448877299e8 + (-0.4453783741e6 + (3257.926478908996 + (-10.82545259305382 + (0.06898058947898353 + 0.4703653406e-4 * x) * x) * x) * x) * x) * x) * x) * x) * x);

%%%% Cuarto intervalo, x >= 20000
$\widetilde{W_{04}}(x)$ =log((1 + x)) - log(1 + log((1 + x))) + (1 + log(1 + log((1 + x))) + (-(1/2) + log(1 + log((1 + x))) ^ 2 / 2 + (-1 / 6 + (-1 + (-0.5 + log(1 + log((1 + x))) / 3) * log(1 + log((1 + x)))) * log(1 + log((1 + x)))) / (1 + log((1 + x)))) / (1 + log((1 + x)))) / (1 + log((1 + x)));

ralta_W0:= piecewise(-exp(-1) <= x and x < 1, $\widetilde{W_{01}}(x)$, -1 <= x and x < 40, $\widetilde{W_{02}}(x)$, 40 <= x and x < 20000, $\widetilde{W_{03}}(x)$, x >= 20000, $\widetilde{W_{04}}(x)$);

Nota: La instrucción *piecewise* se utiliza para concatenar las diferentes aproximaciones especificando el intervalo donde será válido. Esta instrucción se deberá modificar o quitar dependiendo del *software* donde se implemente.