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Artículos científicos

GeoGebra para el aprendizaje de modelación matemática en ingeniería: estudio de caso (modalidad en línea)

GeoGebra for Learning Mathematical Modeling in Engineering: Case Study (Online Modality)

GeoGebra para aprender modelagem matemática em engenharia: estudo de caso (modo online)

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Resumen

En este estudio de caso, el objetivo fue contrastar cuantitativamente y cualitativamente el logro del aprendizaje de modelación matemática (adquisición de lenguaje formal, construcción del modelo, solución e interpretación del modelo y aplicación del modelo) entre un enfoque sistémico que recurre al uso del *software* GeoGebra y un enfoque convencional. Esta investigación tuvo un alcance mixto, descriptivo y parcialmente experimental. Se trabajó, en una modalidad en línea, con una muestra no probabilística de 130 estudiantes de ingeniería, segmentados en un grupo control y otro experimental. Respecto a los resultados obtenidos, se encontró que al final de las tres etapas del estudio, en una escala de 0 a 10, el grupo control logró un incremento en desempeño de 3.01 y que, habiendo transcurrido cuatro meses, se perdieron 1.95 puntos. Por su parte, el grupo experimental obtuvo un incremento en desempeño de 4.66 y después de cuatro meses tuvo una reducción de 0.94 puntos. El incremento real en desempeño de la etapa de diagnóstico al punto final fue de 1.06 para el grupo control y de 3.72 para el grupo experimental. En términos cuantitativos, esto representa una diferencia absoluta estadísticamente significativa entre el grupo control y el experimental de 26 puntos porcentuales, lo que equivale a una diferencia relativa de 250.94 % entre el diagnóstico y la etapa realizada a cuatro meses de la intervención. El incremento en desempeño fue mayor en el grupo experimental. Los resultados indican que el enfoque sistémico de aprendizaje propuesto asistido con GeoGebra pudiera incidir de forma positiva en el desempeño en modelado matemático.

Palabras clave: didáctica, modalidad en línea, simulación, transferencia de registros semióticos, Vigotsky, visualización matemática.

Abstract

In this case study, the objective was to contrast quantitatively and qualitatively the achievement of mathematical modeling learning (acquisition of formal language, construction of the model, solution and interpretation of the model and application of the model) between a systemic approach that used the GeoGebra software and a conventional approach. This research had a mixed, descriptive and partially experimental scope. It was carried out in an online modality with a non-probabilistic sample of 130 engineering students, segmented into a control group and an experimental group. Regarding the results obtained, it was found that at the end of the three stages of the study, on a scale of 0 to 10, the control group achieved an increase in performance of 3.01 and that, after four months, 1.95 points were lost. While the experimental group obtained an increase in performance of 4.66 and after four months it had a reduction of 0.94 points. The actual increase in performance from the diagnostic stage to the end point was 1.06 for the control group and 3.72 for the experimental group. In quantitative terms, this represents a statistically significant absolute difference between the control and experimental groups of 26 percentage points, which is equivalent to a relative difference of 250.94% between the diagnosis and the stage carried out four months after the intervention. The increase in performance was greater in the experimental group. The results indicate that the proposed systemic learning approach assisted with GeoGebra could have a positive impact on performance in mathematical modeling.

Keywords: didactics, online modality, simulation, semiotic register transfer, Vigotsky, mathematical visualization.

Resumo

Neste estudo de caso, o objetivo foi contrastar quantitativa e qualitativamente a realização da aprendizagem da modelagem matemática (aquisição da linguagem formal, construção do modelo, solução e interpretação do modelo e aplicação do modelo) entre uma abordagem sistêmica que recorre à uso do software GeoGebra e uma abordagem convencional. Esta pesquisa teve um escopo misto, descritivo e parcialmente experimental. Trabalhamos, na modalidade online, com uma amostra não probabilística de 130 estudantes de engenharia, segmentados em grupo controle e grupo experimental. Em relação aos resultados obtidos, verificou-se que ao final das três etapas do estudo, em uma escala de 0 a 10, o grupo controle obteve um aumento de desempenho de 3,01 e que, após quatro meses, 1,95 pontos foram perdidos. Por sua vez, o grupo experimental obteve um aumento de desempenho de 4,66 e após quatro meses teve uma redução de 0,94 pontos. O aumento real no desempenho da fase de diagnóstico até o ponto final foi de 1,06 para o grupo controle e 3,72 para o grupo experimental. Em termos quantitativos, isso representa uma diferença absoluta estatisticamente significativa entre os grupos controle e experimental de 26 pontos percentuais, o que equivale a uma diferença relativa de 250,94% entre o diagnóstico e o estágio realizado quatro meses após a intervenção. O aumento no desempenho foi maior no grupo experimental. Os resultados indicam que a abordagem de aprendizagem sistêmica proposta auxiliada pelo GeoGebra pode ter um impacto positivo no desempenho em modelagem matemática.

Palavras-chave: didática, modalidade online, simulação, transferência de registros semióticos, Vygotsky, visualização matemática.

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Introduction

This research addressed a case study of the qualitative and quantitative impact of the incorporation of GeoGebra, dynamic geometry software (DGS), in the teaching-learning process of the topic of mathematical modeling. There was a non-probabilistic sample of 130 initial level engineering students, which was divided equally into a control group and an experimental group. GeoGebra was selected for its didactic design, characterized by integrating two semiotic registers, one analytical/algebraic and the other graphic. This property could contribute to the development of visualization skills, critical in understanding mathematical concepts. This section integrated background, justification, problem statement, objectives and working hypothesis.

The contributions of this work result from the integration of a DGS, the approach of a pertinent but scarcely studied thematic content such as mathematical modeling, the field of engineering, the context of online modality and the mixed research approach. Based on the systematic review conducted from 2011 to 2021, no previous studies with these five combined elements were identified.

In reference to the impact of technology on mathematics instruction, Young (2017) conducted a meta-analysis of studies whose purpose was to determine the cumulative effects of technology on student mathematics performance on achievement tests. One of the findings suggested that technology-assisted math instruction has a moderate but statistically significant cumulative positive effect.

Similarly, Chan and Leung (2014) conducted a meta-analysis of studies from 2001 to 2013 using the ProQuest, JSTOR, ERIC, PsycINFO, and SwetsWise databases to determine the effects of DGS-assisted instruction on mathematical performance on tests. performance of students in grades K-12 compared to conventional instruction. Results indicated that DGS-supported instruction has a positive and statistically significant impact on student math performance.

According to Chan and Leung (2014) and Samur (2015), the DGS allow students to visualize concepts, build relationships, discover patterns and their generalization, perform geometric proofs, and develop their skills such as problem solving and creative thinking. GeoGebra is located within this type of DGS.

Gutiérrez, Prieto and Buitrago (2017) pointed out that "this tendency to use the technological tools available when working mathematically is evidence of the students' ability to adapt the GeoGebra tools to the simulation situation in the scene" (p. 62).

Regarding some of the properties and potentialities of GeoGebra for the teaching and learning of mathematical content, Bayazit and Aksoy (2010) considered the following:

> The manipulation performed in one of these windows is immediately updated in the other. This feature of GeoGebra allows students to understand the conceptual links between representations of a mathematical concept and eventually promotes their vertical growth (depth of understanding) and horizontal growth (development of knowledge through representations) of this concept. (p. 95).

And Jiménez and Jiménez (2017) stated that

> The incorporation of dynamic environments, in particular GeoGebra, in the training of mathematics teachers favors the construction of meaningful, operational and structured mathematical knowledge, which allows them to move easily between symbolic, numerical, graphic and analytical representation systems (p. 12).

On GeoGebra in particular, some research has been developed with different objectives. Arbain and Schukor (2015) focused on the influence of using GeoGebra on students' academic success and their attitudes towards mathematics. For their part, Murni, Sariyasa and Ardana (2017), in agreement with Jacinto and Carreira (2016), analyzed the influence of GeoGebra on the development of specific mathematical skills such as problem solving. Nobre et al. (2016) and Poon (2018) made contributions on how to use GeoGebra in the teaching of specific subjects within the broad field of mathematics. Zetriuslita, Nofriyandi and Istikomah (2021) carried out a work with a mixed scope that focused on identifying improvements in self-efficacy and self-regulation through GeoGebra-based teaching in university mathematics students. They concluded that GeoGebra-based teaching was effective in increasing students' self-efficacy and self-regulation. Báez, Pérez and Blanco (2018) conducted a study on the use of mathematical assistants such as GeoGebra and SketchPath in learning differential calculus and concluded the following:

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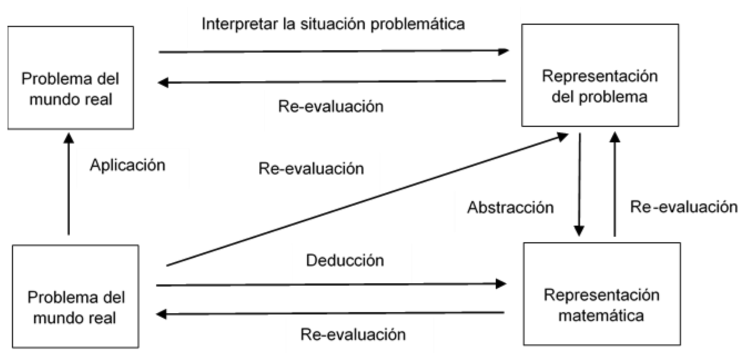
> The experimental validation allowed to demonstrate that the proposal leads to a significant improvement of the students, in relation to the mathematical language and to the conceptual applications in the differential calculus, identifying and manipulating the movement of the variables, which acquired their own resources to carry out register transfers. semiotics and made the concept independent of its representations, all of which contributed notably to its conceptual formation, where the mathematical assistants constituted an adequate didactic scenario, not only as a tool for mathematical activity but also as an element of motivation for the students. (p. 24).

Regarding the use of GeoGebra for the construction of simulations and representation of real phenomena, Villamizar (2020) suggested that it can help in experimentation, obtaining data on behavior patterns, visualization and manipulation of real phenomena through simulation.

In another order of ideas, one of the core concepts of this work is that of mathematical modeling. It is not a new item, however, it is still current. “Mathematical modeling is widely studied in many countries such as Germany, Turkey, and Australia, and is gradually becoming the leading research in mathematics education in the United States” (Been, 2016, p. 7).

Figura 1. Un proceso general de modelación matemática



Fuente: Ashim y Sahin (2019, p. 253)

As can be seen in figure 1, mathematical modeling is not a linear process, but a cycle that starts from the real world and returns to it and involves cognitive processes.

Several works have pointed out the inconvenience of disassociating the learning of mathematics from real or application problems and, therefore, its inadequacy for solving problems in everyday, work or professional contexts (Daher and Shahbari, 2015; Huincahue, Borromeo and Mena, 2018; Jung, Stehr and He, 2019; Pertamawati and Retnowati, 2019; Rodríguez and Quiroz, 2016).

In addition to this, Shabhari and Peled (2017), Schukajlow, Kolter and Blum (2015), Doerr, Arleback and Castello (2014) and Plaza (2016) carried out studies in which the level of learning achievement of content and objects was compared. mathematics, as well as autonomy, decision-making, planning and structuring skills, among others, from the approach to mathematics through modeling in contrast to conventional methods. In the end, they concluded that the students benefited from starting from contextual problems and moving towards formal mathematical concepts.

Other authors such as Kurniadi, Darmawijoyo and Pratiwi (2020), Jacobs and Durandt (2017), Yenmez, Erbas, Cakiroglu, Cetinkaya and Alacaci (2018) and Zeytun, Cetinkaya and Erbas (2017), among others, raised the existence of a discrepancy between actual educational practice compared to theory and official educational programs that incorporate mathematical modeling. They pointed out the relevance of strengthening pedagogical and content teaching competencies with respect to mathematical modeling so that this empowerment is implemented in educational practice.

The following structural elements were considered in this study. The independent variable was the applied didactic proposal that had two contexts: a) synchronous videoconference and instructional design based on Gagné and Briggs and b) synchronous videoconference, instructional design based on Gagné and Briggs and DGS GeoGebra, didactically articulated with a systemic approach. For its part, the dependent variable was the level of quantitative and qualitative performance in the achievement of learning mathematical modeling (acquisition of formal language, construction of the model, solution and interpretation of the model, and application of the model).

In this study, the hypothesis had this approach: in a context of online modality, if the teaching-learning process of mathematical modeling is implemented with a systemic approach, considering a) appropriation of resources for the transfer of semiotic records assisted by GeoGebra, b) acquisition of formal language assisted by GeoGebra, c) transition from conversational language to formal mathematical language and d) interpretation and manipulation of formal mathematical language assisted by GeoGebra, considering these elements, we said, then a quantitative and qualitative increase in achievement will be generated. learning of mathematical modeling (acquisition of formal language, construction of the model, solution and interpretation of the model and application of the model) compared to using a conventional approach.

The objective was to contrast quantitatively and qualitatively the achievement of mathematical modeling learning (formal language acquisition, model construction, model solution and interpretation, and model application) in a GeoGebra-assisted systemic approach in relation to a conventional approach.

Methodology

The research had a mixed scope (quantitative-qualitative), descriptive and partially experimental. We worked with a non-probabilistic sample of engineering students from the Autonomous University of the State of Morelos (UAEM) in Mexico, made up of 130 elements, segmented into two groups of 65 elements each, one control and one experimental. The sample was drawn from a homogeneous segment of an entry-level study population. This case study was conducted online. An instructional design based on the Gagné and Briggs model was used as a repository of learning content and evidence, due to its comprehensive 14-step structure, methodologically based on systems theory. Synchronous videoconferencing was used in several sessions, which totaled 12 hours. Taking into account that in an online modality it is possible to lose control of factors that directly affect performance results, some rudimentary mechanisms were implemented, such as personalizing the evaluations with a multiplying factor based on the Unique Population Registry Key (CURP) , strictly limit start, end and delivery times, keep the camera on throughout the process, solve the evaluation instruments by handwriting, do not use a calculator and document in detail all the operations carried out. There was documented, constant and random supervision of the students' writing. Although it would be desirable to incorporate technological elements such as the strict timed identification of internet protocol address access and physical addresses of the network cards of all the electronic devices used and associate them with the videoconference and the technological platform, for the purposes of this research There were no such applications. It should be noted that the previous elements, without exception, were common to the entire sample.

Tabla 1. Caracterización de la muestra estudiada

Grupo control = 65 elementos	Grupo experimental = 65 elementos
Videoconferencias sincrónicas, diseño instruccional	Videoconferencias sincrónicas, diseño instruccional y enfoque sistémico asistido por GeoGebra

Fuente: Elaboración propia

The variation between the control and experimental groups was that in the latter a mathematical modeling teaching-learning process was implemented with a systemic approach that took into account the following: a) appropriation of resources for the transfer of semiotic records assisted by GeoGebra, b) acquisition of formal language assisted by GeoGebra, c) transition from conversational language to formal mathematical language and d) interpretation and manipulation of formal mathematical language assisted by GeoGebra. Table 1 shows the characterization of the studied sample. While figure 2 shows a prototype problem of work with both groups. It should be noted that mathematical modeling is extensive and varied.

Figura 2. Ejemplo de problemas de modelación matemática abordados

La empresa "Maquiladora, S.A. de C.V" requiere producir dos tipos de cable: normal y premium. En el almacén dispone de 500 kg de alambre A, 300 kg de alambre B y 108 kg de alambre C. Se está considerando la producción diaria, para obtener un metro de cable normal se necesitan 125 gr de A, 150 gr de B y 72 gr de C; para producir un metro de premium por día se necesitan 200 gr de A, 100 gr de B y 27 gr de C. El normal aporta una utilidad de $ 4000 y el premium de $ 5000 el metro. Si se debe obtener la máxima utilidad, ¿cuántos metros de normal y premium se deben manufacturar?

Insumos	Producto final (1 metro de normal) gramos	Producto final (1 metro de premium) Gramos	Disponibilidad gramos
Alambre A	125	200	500,000
Alambre B	150	100	300,000
Alambre C	72	27	108,000
Utilidad unitaria	4000	5000	

Modelo matemático
x1= cantidad de metros de cable normal a producir
x2= cantidad de metros de cable premium a producir

Max Z = 4000x1 + 5000x2
125x1 + 200x2 <= 500,000
150x1 + 100x2 <= 300,000
72x1 + 27x2 <= 108,000

Para toda xi >=0, i ? {1,2}

Preguntas:

1.-¿Cuántos metros de cable normal y premium le recomienda a la empresa producir y vender para maximizar sus utilidades ?¿con base en qué le sugiere esa decisión?
2.-¿Existe un único conjunto de decisiones de producción a tomar o más de uno que reporte la misma utilidad?
3.-¿Cuál es la utilidad máxima que puede esperar la empresa dadas las disponibilidades de recursos en los almacenes?
4.-¿En qué recurso se ubica "el cuello de botella" de la empresa y cómo afecta la utilidad máxima esperada?

Fuente: Elaboración propia

A battery was designed with different versions of evaluation instruments for three problems with 51 mathematical modeling reagents that could be solved with graphic or analytical procedures. These were organized based on Bloom's cognitive levels, as shown in Table 2.

Tabla 2. Clasificación de reactivos del instrumento de evaluación

Niveles cognitivos	Reactivos	Porcentaje
Comprender	3	5.8 %
Aplicar	9	17.64 %
Analizar	9	17.64 %
Evaluar	18	35.29 %
Crear	12	23.52 %
Total	51	100.00 %

Fuente: Elaboración propia

The evaluation instrument was applied in three moments: a) diagnosis, b) intervention and c) four months after the intervention. In the second application, the learning potential of the sample was estimated considering 12 hours of training. The third application had the purpose of determining a level of consolidation of learning over time.

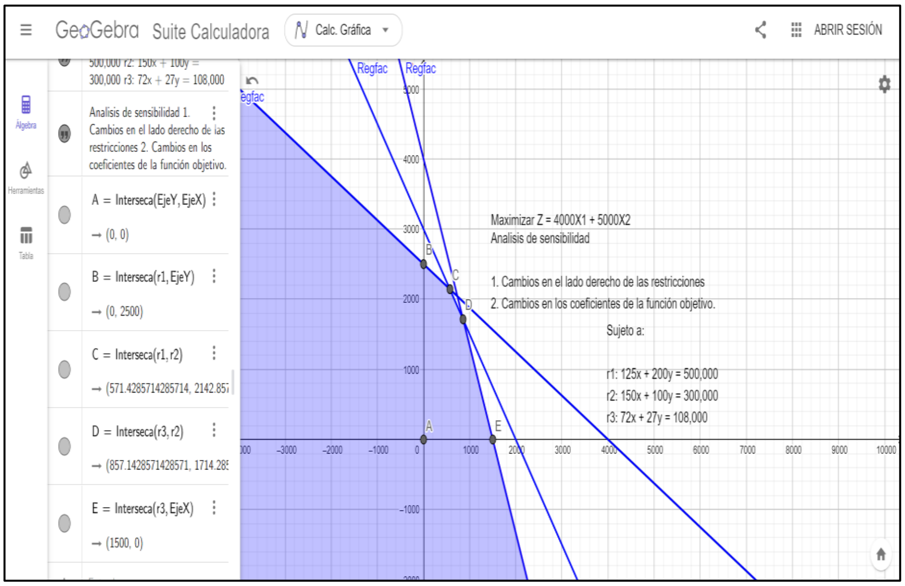
The achievement of mathematical modeling learning was defined in both cases by the acquisition of formal language, construction of the model, solution and interpretation of the model, and application of the model, which were considered in the design of the battery of evaluation instruments.

The conventional intervention consisted of dictating synchronous videoconferences on the solution of mathematical modeling prototype problems, using an editor as an electronic whiteboard to propose the solution consisting of the system of equations that constitute the mathematical model, composed of an objective function, a set of restrictions which can be equalities or inequalities and the non-negativity constraints. It was also shown in the exhibition how to solve the models by graphic and analytical methods. The students solved some problems proposing the mathematical model, the values of the variables and the objective function. Likewise, sensitivity analysis problems were included in which it was determined how one element of the model was modified when another was altered, for example, how the objective function changed when a coefficient associated with a variable was altered.

In the intervention of the experimental group, synchronous videoconference was also used as a means of communication rather than exposure. The implemented methodology was composed of the following steps: a) appropriation of resources for the transfer of semiotic registers assisted by GeoGebra, b) acquisition of formal language assisted by GeoGebra, c) transition from conversational language to formal mathematical language and d) interpretation and manipulation of formal mathematical language assisted by GeoGebra.

In the case of the experimental group, the prototype problems were also used as working information. Regarding the appropriation of resources for the transfer of semiotic records assisted by GeoGebra, it consisted of dictating a 20-minute videoconference to show students how to enter data into the software, exemplifying all the elements of the model and its sensitivity analysis. Subsequently, the students had three personalized exercises in which they were required to enter input data for a problem and its model. They were also asked to perform sensitivity analyzes graphically. Figure 3 shows one of the students' works considering graphical and analytical records of mathematical concepts such as feasible region, polytope and vertices of the polytope or points of intersection of the restrictions.

Figura 3. Conceptos matemáticos de *región factible*, *politopo* y *puntos de intersección* trabajados por los estudiantes con asistencia de GeoGebra



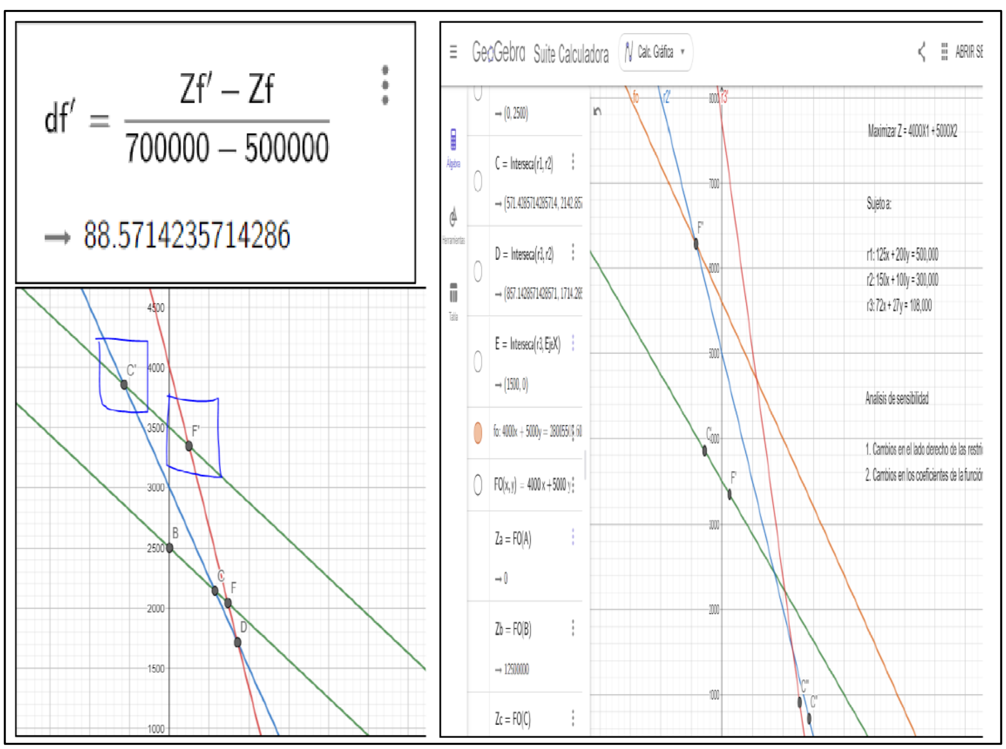
Fuente: Elaboración propia

Regarding the acquisition of formal language assisted by GeoGebra, care was taken to use mathematical language in the initial videoconference, associating each term with its respective graphic or analytical record of GeoGebra to facilitate its understanding and acquisition. GeoGebra mathematical functions were also capitalized on, for example, associating the intersect function with the vertices of the polytope that represent the decision combinations that must be evaluated to find the optimal solution. It was graphically and analytically shown that any combination of variables within the feasible zone is suboptimal, which is properly a mathematical theorem based on the theory of differential calculus. The students were rigorously asked to submit their work avoiding using conversational language. It should be noted that the GeoGebra data entry syntax is a strictly technical language.

In general, in all areas of mathematics, the transition from conversational language to formal mathematical language is characterized by requiring a developed capacity for abstraction. This ability implies a level of difficulty of the highest cognitive level, based on Bloom. By its nature, it is difficult to assist the development of this capability with technology. In the case of this specific content of mathematical modeling, the language transition was limited to an analytical reading of the text of the problem to identify the elements of the model such as decision variables, coefficients of the objective function, resource utilization rates of the decision variables, levels of availability of resources and congruence in the units of measurement. This allowed specifying the construction of equations of the objective function and the restrictions that constitute the formal language for the case of mathematical modeling of this type. Finally, emphasis was placed on determining the type of programming, integer or linear, which defines the non-negativity constraints. It should be noted that this strategy would not work to move towards a formal language required by proofs of pure mathematics, for example.

Regarding the interpretation and manipulation of formal mathematical language assisted by GeoGebra, once the original model was solved, modifications were made to the coefficients of the objective function and the availability to apply sensitivity analysis. GeoGebra made it possible to immediately graphically visualize the impact of the proposed alterations to the model, as shown in Figure 4.

Figura 4. Conceptos matemáticos de análisis de sensibilidad (cambios en coeficientes de función objetivo y lado derecho de restricciones, precios sombra y dualidad) trabajados por los estudiantes con asistencia de GeoGebra



Fuente: Elaboración propia

Results

In the diagnostic phase, 6,630 responses from 130 study subjects were evaluated, divided into a control group and an experimental group, with 65 elements each. Post-intervention, 6630 equivalent exercise responses were also evaluated from the same 130 study subjects segmented in the same way. In the evaluation four months after the intervention, the locatable elements were evaluated, so that there were 62 members of the control group and 63 of the experimental group out of a total of 65 in each of them, that is, three were missing and two of them were missing. each group, respectively. In this phase, 3,162 responses from the control group and 3,213 from the experimental group were evaluated, giving a total of 6,375 qualified responses for this third stage. Table 3 summarizes the evaluations carried out in the three stages of the project.

Tabla 3. Número de respuestas evaluadas por etapa del proyecto y por grupo de investigación

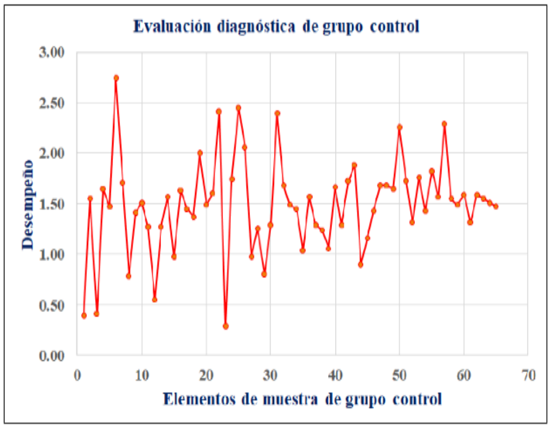
Número de respuestas evaluadas por etapa del proyecto y por grupo de investigación							
Etapa	1) Diagnóstico		2) Intervención		3) Cuatro meses		Total de respuestas evaluadas
	Elementos	Diagnóstica	Elementos	Intervención	Elementos	A cuatro meses de la intervención	
Grupo control	65	3315	65	3315	62	3162	9792
Grupo experimental	65	3315	65	3315	63	3213	9843
Total	130	6630	130	6630	125	6375	19635

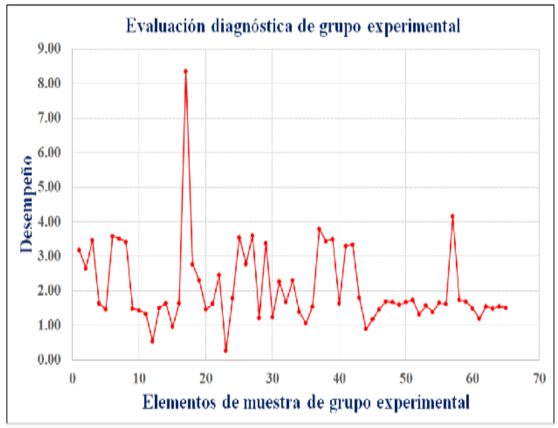
Fuente: Elaboración propia

Results of the diagnostic evaluation

Figure 5 shows the detailed results of the diagnostic evaluation of the control and experimental groups, respectively, each of 65 elements.

Figura 5. Desempeño diagnóstico de los grupos control y experimental





Fuente: Elaboración propia

Tabla 5. Estadística descriptiva: etapa diagnóstica

Grupo	Promedio	Desviación estándar	Mediana
Grupo control	1.51	1.31	1
Grupo experimental	2.14	1.77	2
Diferencias	0.63	0.46	1
Promedio general	1.82	1.54	N. a.

Fuente: Elaboración propia

Figure 5 and Table 5 show that both the control and experimental groups presented statistically significant variability or dispersion. The arithmetic mean performance of the control group was 1.51 with a standard deviation of 1.31. The arithmetic mean performance of the experimental group was 2.14 with a standard deviation of 1.77. This means that 68.2% of the control group performed within the closed interval 0.21, 2.82 considering one standard deviation based on the normal probability distribution. These metrics mean that 95.4% were in the interval 0, 4.13, considering two standard deviations, or 99.6% performed in the interval 0, 5.94, if three standard deviations are applied. For its part, this same percentage of the experimental group performed within the closed interval 0.37, 3.91, considering one standard deviation. It was observed that 95.4% performed in the interval 0, 5.68, considering two standard deviations. A proportion of 99.6% had scores in the interval 0, 7.45, applying three standard deviations. The difference between the means was 0.63 points, being higher for the experimental group. However, this difference could be negligible on the 0, 10 scale.

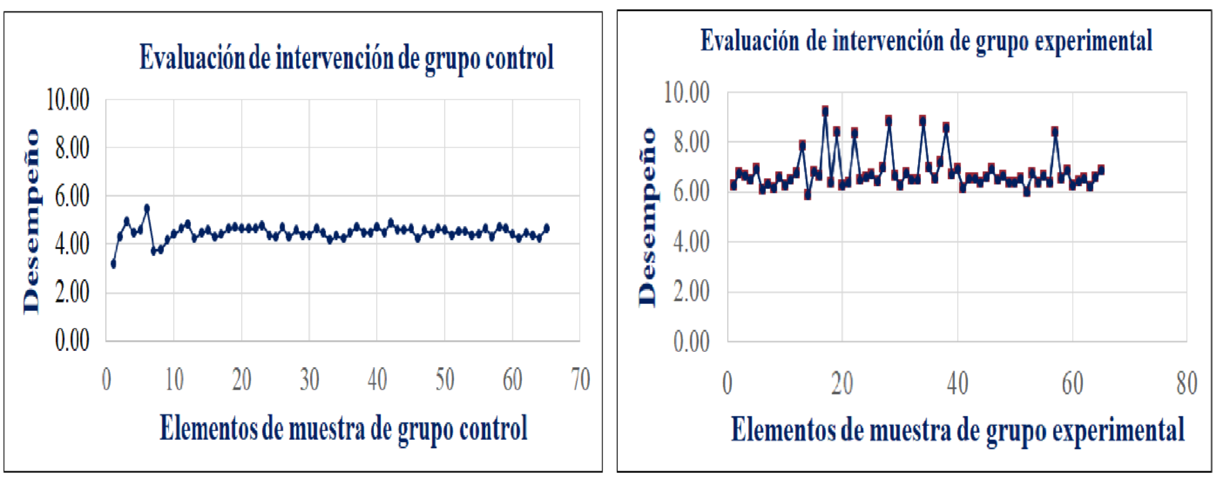
Although the arithmetic mean of the experimental group is higher, its standard deviation is also higher, that is, in comparative terms the experimental group would have a higher relative performance, however , presents greater dispersion or variability, which is not desirable.

Intervention evaluation results

Once the intervention lasted 12 hours, 130 evaluations were applied with 51 reagents each, 65 of these were for the control group and another 65 for the experimental group. In total, 6630 responses from both groups were evaluated. Figures 6 and 7 and Table 6 show the final results of both research groups.

In figure 6 and table 6 it can be seen that the control group presents less variability than the experimental group. It is shown that the arithmetic mean performance of the control group was 4.52 with a standard deviation of 1.12. The arithmetic mean of the experimental group was 6.80 with a standard deviation of 1.76. This meant that 68.2% of the control group performed within the closed interval 3.4, 5.64 considering one standard deviation. These metrics imply that 95.4% were in the interval 2.28, 6.76 applying two standard deviations.

Figura 6. Desempeño de la intervención de los grupos control y experimental



Fuente: Elaboración propia

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Tabla 6. Estadística descriptiva: etapa de intervención

Grupo	Promedio	Desviación estándar	Mediana
Grupo control	4.52	1.12	4
Grupo experimental	6.80	1.76	7
Diferencias	2.28	0.64	3
Promedio general	5.66	1.44	N. a.

Fuente: Elaboración propia

If three standard deviations are taken into account, a proportion of 99.6% performed in the interval 1.16, 7.88. For its part, this same percentage of the experimental group performed within the closed interval 5.04, 8.56, with one standard deviation. Applying two standard deviations, 95.4% performed in the interval 3.28, 10. A total of 99.6% had scores in the interval 1.52, 10, if three standard deviations are considered. The sample was taken from a homogeneous segment of a study population at the initial engineering level. The difference between the means is 2.28 points, being higher for the experimental group. The difference of the standard deviation is 0.64, being lower than that of the control group.

Performance results four months after the evaluation of the intervention

The third phase was carried out four months after the intervention. 125 evaluations were applied with 51 reagents each, 62 of these were for the control group and another 63 for the experimental group. Three elements of the control group and two elements of the experimental group were not locatable. In total, 6,375 responses from both groups were evaluated at this stage. Figures 7 and 8 and Table 7 show the final results of both research groups.

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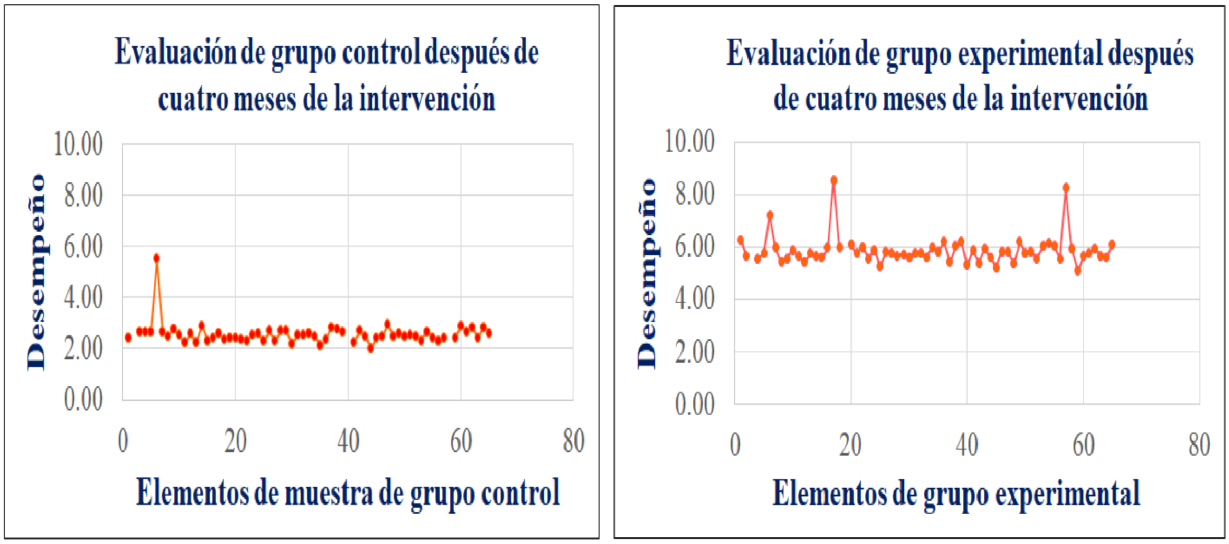
Figura 7. Desempeño de los grupos control y experimental, después de cuatro meses de la intervención



Fuente: Elaboración propia

Tabla 7. Estadística descriptiva: etapa cuatro meses posteriores a la evaluación de la intervención

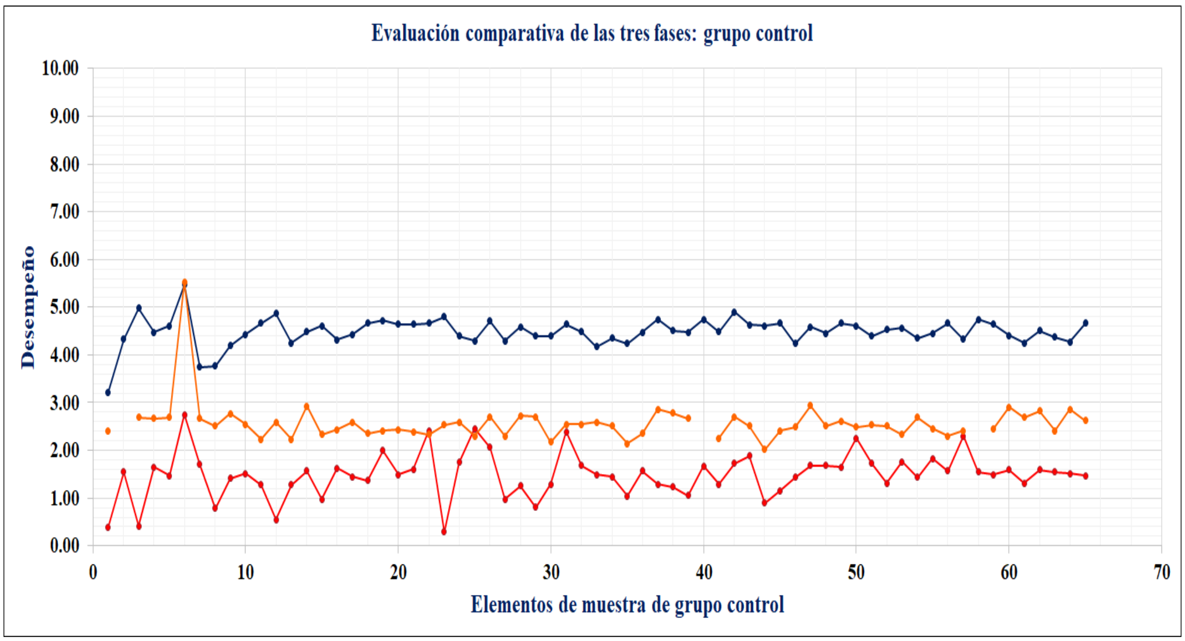
Grupo	Promedio	Desviación estándar	Mediana
Grupo control	2.57	1.73	3
Grupo experimental	5.86	2.03	6
Diferencias	3.29	0.3	3
Promedio general	4.21	1.88	N. a.

Fuente: Elaboración propia

Figure 7 and Table 7 show that, at least in graphical terms, the variability or dispersion has been reduced in this last stage. Table 7 shows that the arithmetic mean performance of the control group was 2.57 with a standard deviation of 1.73. The arithmetic mean performance of the experimental group was 5.86 with a standard deviation of 2.03. This means that 68.2% of the control group performed within the closed interval 0.84, 4.3 considering one standard deviation.

These metrics mean that 95.4% were in the interval 0, 6.03, considering two standard deviations, or that 99.6% performed in the interval 0, 7.76, considering three standard deviations. For its part, this same percentage of the experimental group performed within the closed interval 3.83, 7.89, taking one standard deviation. The proportion of 95.4% performed in the interval 1.8, 9.92, applying two standard deviations. It was observed that 99.6% had scores in the interval 0, 10, taking three standard deviations into consideration. The difference between the means is 3.29 points, being higher for the experimental group. The difference of the standard deviation is 0.3, being lower than that of the control group.

Figura 8. Comparativo de evaluación de desempeño de las tres etapas de evaluación: grupo control. Rojo: etapa de diagnóstico, azul: etapa de intervención, naranja: etapa a cuatro meses de la intervención



Fuente: Elaboración propia

Figures 8, 9, 10 and tables 8 and 9 show the concentrated results of performance in mathematical modeling of the control and experimental groups in the three stages. With this information, changes in the level of performance of each individual element of the study sample and also of group behavior can be identified. For example, considering the performance in average mathematical modeling per stage for the control group, it is observed that it starts from 1.51 in stage one, rises to 4.52 in stage two and drops to 2.57 in stage three.

In another order, the experimental group starts from a performance of 2.14 in stage one, rises to 6.80 in stage two and drops to 5.86 in stage three. In this work, a metric is proposed that considers the change in the level of progress and the time in which it was achieved. This metric should be conceptualized as "Quantitative change in performance level/Amount of time spent in hours". Below is a table that summarizes those changes in performance levels through the three stages in which the case study was conducted.

Tabla 8. Medidas avance de la fase diagnóstico, intervención y cuatro meses posteriores a esta

Grupo	Avance en desempeño de la fase de diagnóstico a la de intervención	Avance en desempeño de la fase de intervención a cuatro meses de esta	Nivel de desempeño final	Avance en desempeño real final sobre tiempo total invertido de entrenamiento/Intervención
Control	(4.52-1.51) = 3.01	(2.57-4.52) = -1.95	2.57	1.06/12h
Experimental	(6.80-2.14) = 4.66	(5.86-6.80) = -0.94	5.86	3.72/12h

Fuente: Elaboración propia

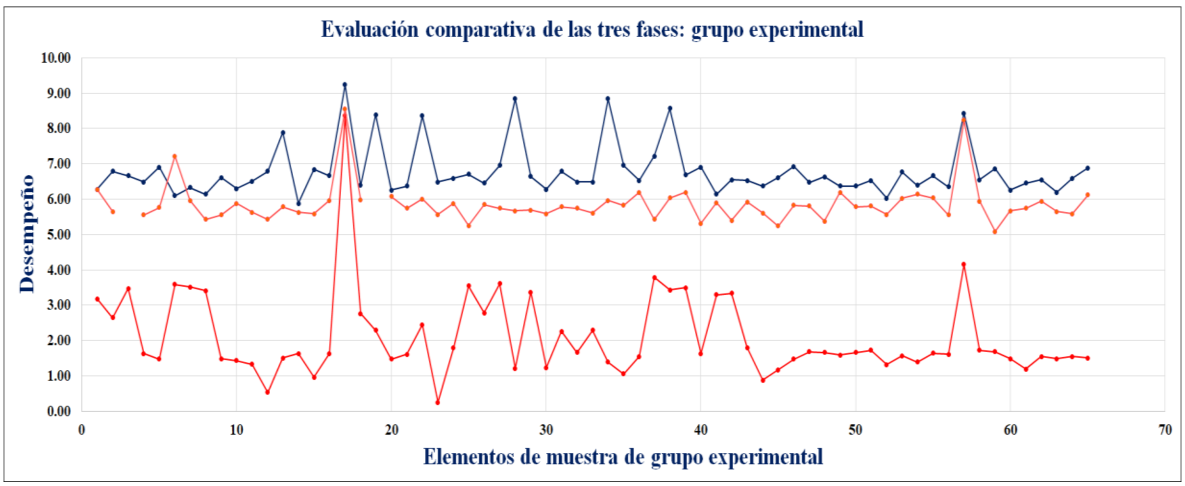
Taking into consideration the time invested of 12 hours with both groups, one interpretation is that the control group treated with the conventional approach achieved an increase in performance of 3.01 points/12 hours and that, after four months, 1.95 points were lost; thus, it ended with a real performance of 2.57/12 hours on a scale from 0 to 10. For its part, the experimental group, treated with the systemic approach, achieved an increase in performance of 4.66 points/12 hours and after four months had a reduction of 0.94 points, for which it obtained a final grade of 5.86/12 hours on the absolute scale of 0 to 10. In reference to the final performance, an absolute difference between the two groups of 3.29 or relative of 128.01% is observed . If the change or progress achieved in relation to the diagnostic stage is measured, an absolute difference between the two groups of 2.66 is observed, equivalent to a relative difference of 250.94%. Some results are shown below for comparison purposes of the three stages.

Tabla 9. Medidas de tendencia central y de dispersión de la variable desempeño en modelación matemática en las tres etapas de la investigación

Etapa	Etapa uno: diagnóstico			Etapa dos: intervención			Etapa tres: a cuatro meses de intervención		
	Promedio	Desviación estándar	Mediana	Promedio	Desviación estándar	Mediana	Promedio	Desviación estándar	Mediana
Grupo control	1.51	1.31	1	4.52	1.12	4	2.57	1.73	3
Grupo experimental	2.14	1.77	2	6.80	1.76	7	5.86	2.03	6

Fuente: Elaboración propia

Figura 9. Comparativo de evaluación de desempeño en modelación matemática de las tres etapas de evaluación: grupo experimental. Rojo: etapa de diagnóstico, azul: etapa de intervención, naranja: etapa a cuatro meses de la intervención



Fuente: Elaboración propia

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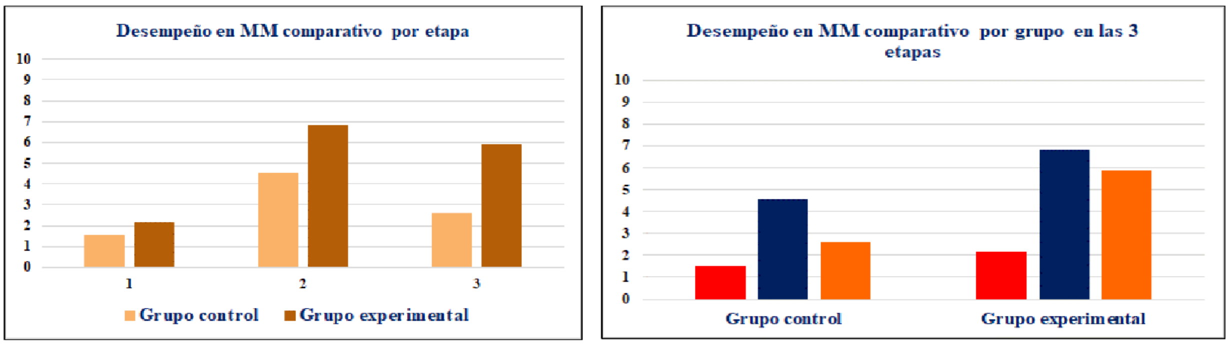
Figura 10. Desempeño en modelación matemática comparativo por grupo en las tres etapas. Rojo: etapa de diagnóstico, azul: etapa de intervención, naranja: etapa a cuatro meses de la intervención



Fuente: Elaboración propia

Discussion

This article has tried to show the potential impact of the implementation of a systemic approach: a) appropriation of resources for the transfer of semiotic registers assisted by GeoGebra, b) acquisition of formal language assisted by GeoGebra, c) transition from conversational language to formal mathematical language and d) interpretation and manipulation of formal mathematical language assisted by GeoGebra), compared to a conventional approach in an online context. The research had three stages: a) diagnosis, b) intervention and c) four months after the intervention with the aim of having a metric of consolidation of learning over time. From the diagnostic to the intervention phase, the control group increased their performance by 3.01/10 (30.1 percentage points) and the experimental group by 4.66/10 (46.6 percentage points). Four months after the intervention phase, there was a drop in performance of -1.95/10 (19.5 percentage points) in the control group and -0.94/10 (9.4 percentage points) in the experimental group. The final performance of the control group was 2.57/10 (25.7 percentage points) and the experimental group was 5.86/10 (58.6 percentage points). The actual increase in performance from the diagnostic stage to the endpoint of the study was 1.06/10 (10.6 percentage points) for the control group and 3.72/10 (37.2 percentage points) for the experimental group. In quantitative terms, this represents a statistically significant absolute difference between the control and experimental groups of 2.66/10, that is, 26 percentage points, which is equivalent to a relative difference of 250.94% between the diagnostic stage and the stage performed at four months of the intervention. The increases in performance are greater in the experimental group and the intermediate drop caused by the lack of practice in the use of knowledge is practically half in the experimental group compared to the control group, which is statistically significant. These metrics must be associated with a time investment with students limited to 12 hours.

In quantitative terms, these statistical results are in agreement with the findings of Kusuman et al. (2021), who conducted a meta-analysis on the effectiveness of dynamic geometry application software in mathematical learning processes. They conducted a study based on a random effects model. Among their findings, they considered that the statistical results obtained were sufficient to demonstrate that the use of dynamic geometry systems in mathematical learning had a high positive impact compared to conventional methods. Their results show that the arithmetic mean in performance of the students who had a treatment with dynamic geometry systems exceeded up to 84% the level of mathematical achievement over those who had a treatment of conventional classes, considering that they started from comparable initial levels of mathematical knowledge. . The pooled effect size they found was 1.07, which is statistically high. They compared their results with another meta-analysis, and although their analysis only considered a tenth of the number of studies as the other meta-analysis, regardless of the size of the study sample, the trend is maintained, since the pooled effect size was 1.02. They concluded that their study and other related studies show that the use of dynamic geometry systems in mathematical learning can improve and have a very high effect on students' mathematical abilities. Among their findings, they also found that there was greater effectiveness at the upper and upper middle level compared to the basic level. The statistically significant difference in performance between the control and experimental groups is coincidental.

In qualitative terms, the results are in line with what was proposed by Zetriuslita et al. (2021), who suggested that instruction assisted by GeoGebra can help improve skills in mathematical language, critical thinking, conceptualization and procedural development thanks to the advantages that characterize it regarding the demonstration and visualization of concepts. mathematicians Among their findings, they also pointed out that having developed critical thinking has a direct and positive correlation with the achievement of learning results. At the end of the intervention, the students expressed that GeoGebra had motivated them to solve problems autonomously (self-regulation and self-efficacy).

In terms of the impact on specific cognitive processes and student attitudes towards the objects of knowledge observed in this study, the results coincided with those reported by Jacinto and Carreira (2016), who carried out a qualitative study with 13-year-old adolescents focused on in one of the math skills, which is problem solving. They observed that the use of GeoGebra digital technology favored in the student the approach of experimental and exploratory approaches, promoted critical thinking and questioning skills, allowed to diversify didactic strategies and promoted the generation of conjectures. In this same order, the results coincided with that reported by Murni et al. (2017), who carried out a study implementing a GeoGebra-assisted discovery learning model for the development of problem-solving skills and attitude towards mathematics. They worked with a sample of 120 students, divided into two experimental groups and two control groups. They concluded that the use of GeoGebra in discovery learning can improve the attitude towards mathematics and the ability to solve problems because it helps them to visualize problems through their immediate response. In methodological aspects, this study and that proposed by these last authors coincide in comparing a control group with an experimental group, with the difference that these researchers proposed a design with two groups of each category to incorporate more contrast elements.

Mollakuqe, Rexhepi e Iseni (2021) carried out a comparative study in a teaching practice of the properties of the circle with GeoGebra and with conventional didactic strategies. In this case, the thematic content is Euclidean geometry specifically. The authors observed that the use of GeoGebra in teaching facilitated, sped up, made geometry more tangible and concrete, helped students to perceive each figure, increased student interest and activated participation through questions and discussion. In the same order of ideas, in this investigation the students pointed out that GeoGebra allowed them to visualize in a more concrete way the elements of the mathematical model, such as the restrictions, their crossings and the direction of the inequalities. Likewise, in the topic of sensitivity analysis of the mathematical model, they were able to immediately observe in the graphs the effects of the changes in the different coefficients.

One of the most important and useful features of GeoGebra's didactic design is that it allows working with two semiotic representations, one analytical and the other graphic, and dynamically relate them in real time and immediately. This study suggests that this possibility of relating both semiotic registers could be the cause that explains the difference in performance between the experimental group and the control group, considering that this helps to improve the understanding of the mathematical concept and its application. This emerging hypothesis of the study coincided with what was proposed by Mosese and Ogbonnaya (2021), who used GeoGebra for learning trigonometric functions with a focus on the connections between their representations and their interpretation. They reported that there was a statistically significant difference in the average performance of experimental students in a technology-rich environment compared to the control group, and additionally, they observed an increase in motivation derived from collaborative activities. They attributed the improvement in the experimental group to the social constructivist learning environment that encouraged Vygotsky's interaction, guesswork, and knowledge construction. They concluded that GeoGebra was effective in improving students' ability to make connections between different representations and contexts, and the same for the interpretation and analysis of trigonometric functions. They reported that most of the students in this study managed to draw the graphs, which contradicted the results reported by Demir in 2012. They indicated that the experimental group had time to explore, investigate and make conjectures about the properties of the different graphs thanks to the GeoGebra instant feedback. The findings of their study suggested several implications for the teaching and learning of mathematics in general and the trigonometric functions in particular.

One of the most important strengths of this work was the mixed approach, which made it possible to capitalize on the metrics and their statistical analysis and complement it with a qualitative approach to investigate the impact on attitudinal aspects, as well as potential factors that would explain the behavior observed.

One of the limitations of this research was time. The results achieved were in an interval reduced to 12 hours of intervention, however, if more intervals of work with and of the students were considered, it is likely that the impact on performance could be cumulative and considerably greater.

One of the weaknesses observed during the search for information carried out for the construction of the theoretical framework of this work is that no specific studies were located in which GeoGebra was implemented in relation to mathematical modeling and the sample outside of engineering. Nor were any studies identified that have used the proposed systemic approach. These facts would allow comparisons to be made with greater objectivity.

Among the areas of opportunity of this study, it can be pointed out that, although the sample of this research was significantly larger than in all the cases of the investigations reported in the reviewed literature, it is recommended that future works work with a larger sample. size and that strictly complies with representativeness and statistical randomness. Another opportunity would be to consider a design with two control and two experimental groups, as proposed by Murni et al. (2017), which would contribute to the reliability of the results and their interpretation.

Conclusions

Based on the analysis of the results obtained, it can be concluded that, in quantitative terms, the learning achieved through the proposed systemic approach, assisted by GeoGebra, had a moderate positive impact, statistically supported, on the level of learning achievement of students. mathematical modeling in initial level engineering students, in contrast to that obtained through conventional means exclusively.

Among the contributions of this study was the inclusion of an additional stage of measurement of results, which allowed measuring the degree of consolidation of learning over time. All the studies in the literature considered had two phases exclusively, one diagnostic and one interventional. A differential contribution was the mixed scope of this research, since the studies reviewed in the literature were mostly qualitative, a limited number were quantitative, and none integrated both approaches.

In this study, GeoGebra demonstrated effectiveness and efficiency in representing difficult-to-understand mathematical concepts such as those that are part of mathematical modeling. In the experimental group, it contributed to the visualization skill, which is critical in mathematical learning in general, because it focuses on the creation of meanings and contextual interpretations. Visual images or graphic semiotic records constituted an effective mechanism to communicate ideas related to mathematical concepts. The dynamic representations were useful to support the activities of analysis, formulation of models and dynamic changes of visualization of variations in the coefficients of the objective function or the independent terms of the system of linear equations, in the specific case of the mathematical concept of analysis of sensitivity of the mathematical model. GeoGebra allowed students to create and explore different aspects of mathematical model representations. GeoGebra made it easy to represent mathematical concepts in a formal, relational, and instrumental way through its visualization and simulation tools.

Future lines of research

Among the lines of research, it is suggested to strengthen the state of the art regarding the use of GeoGebra for learning mathematical concepts, specifically in engineering. In another order, it is also convenient to carry out a review of the literature on the available DGS alternatives with comparative purposes in their characteristics, their didactic treatment of representations and semiotic register transfers and their effectiveness in learning mathematical concepts at different academic levels. Regarding the learning of mathematical modeling, given its relevance in solving real problems, it is pertinent to review the teaching strategies that have been studied and their effectiveness in achieving mathematical learning. On the other hand, the visualization of mathematical concepts is an ability that needs to be developed in academic engineering programs and it is appropriate to analyze the development of this ability from Vygotsky's conception of language as a means of materializing thought and Duval, who He stated that it is necessary to know at least two different ways of expressing or representing a mathematical object in order to learn and understand that object.

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