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Acercamiento a la comprensión de textos en la resolución de problemas en matemáticas considerando los procesos inductivos y deductivos: el caso de Bobby

Approach to the comprehension of texts in the resolution of problems in mathematics considering the inductive and deductive processes: the case of Bobby

Abordagem da compreensão de textos na resolução de problemas em matemática considerando os processos indutivo e dedutivo: o caso de Bobby

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Resumen

La literatura en educación matemática muestra que la habilidad para resolver problemas de esa área está estrechamente relacionada con la comprensión del texto y los conocimientos previos del estudiante. Además, se ha identificado que aunque un estudiante pueda parecer experimentado en la lectura de textos variados, esto no garantiza su comprensión. Por eso, el objetivo de esta investigación fue analizar los procesos de comprensión textual en problemas matemáticos que involucran texto en un estudiante de 15 años durante un curso de álgebra de bachillerato. Para entender estos procesos se utilizaron elementos teóricos que permitieron identificar sus componentes, en particular las operaciones de segmentación y recontextualización en los procesos inductivo, deductivo y evolutivo subyacentes al fenómeno estudiado.

La metodología utilizada fue cualitativa y se empleó el método de estudio de caso. El estudiante seleccionado presentó compromiso y disponibilidad para explorar actividades que involucraron la comprensión de textos. Los instrumentos utilizados para la recolección de datos incluyeron hojas de trabajo, reportes escritos y una entrevista semiestructurada.

A partir del análisis de lo recabado se identificó que el trabajo del estudiante se caracterizó en un inicio por la exploración del contenido redaccional basado en el proceso inductivo de comprensión, del cual emergieron las unidades de texto implícitas y explícitas identificadas. En el proceso deductivo, se observó que el estudiante trabajó en relación con el significado de las fracciones como parte-todo y como índice comparativo entre dos cantidades o conjuntos de unidades. Asimismo, se destacó la importancia de entender que las fracciones son razones que permiten evidenciar la comparación.

Palabras claves: comprensión textual, operaciones fundamentales, deductivo, inductivo.

Abstract

Literature in mathematics education shows that the ability to solve problems in this field is closely related to a student's understanding of text and prior knowledge. Moreover, it has been identified that even if a student seems experienced in reading various texts, it does not guarantee comprehension. Therefore, the aim of this research was to analyze the processes of text comprehension in mathematical problems that involve text in a 15-year-old student during a high school algebra course. Theoretical elements were used to identify the components of these processes, particularly the operations of segmentation and

recontextualization in the underlying inductive, deductive, and evolutionary processes of the studied phenomenon.

The methodology used was qualitative, and the case study method was employed. The selected student demonstrated commitment and availability to explore activities involving text comprehension. The data collection instruments included worksheets, written reports, and a semi-structured interview.

Based on the analysis of the collected data, it was identified that the student's work was initially characterized by the exploration of the textual content based on the inductive process of comprehension, from which implicit and explicit text units were identified. In the deductive process, it was observed that the student worked in relation to the meaning of fractions as part-whole and as a comparative index between two quantities or sets of units. Additionally, it was emphasized that understanding fractions as ratios allows for the demonstration of comparison.

Keywords: textual comprehension, fundamental operations, deductive, inductive.

Resumo

A literatura em educação matemática mostra que a capacidade de resolver problemas nessa área está intimamente relacionada à compreensão do texto e ao conhecimento prévio do aluno. Além disso, identificou-se que embora um aluno possa parecer experiente na leitura de uma variedade de textos, isso não garante sua compreensão. Portanto, o objetivo desta pesquisa foi analisar os processos de compreensão de texto em problemas matemáticos envolvendo texto em um aluno de 15 anos durante um curso de álgebra no ensino médio. Para a compreensão destes processos foram utilizados elementos teóricos que permitiram identificar as suas componentes, nomeadamente as operações de segmentação e recontextualização nos processos indutivos, dedutivos e evolutivos subjacentes ao fenómeno estudado.

A metodologia utilizada foi qualitativa e foi utilizado o método de estudo de caso. O aluno selecionado apresentou comprometimento e disponibilidade para explorar atividades que envolviam a compreensão de textos. Os instrumentos utilizados para a coleta de dados foram planilhas, relatórios escritos e entrevista semiestruturada.

A partir da análise dos dados coletados, identificou-se que o trabalho do aluno caracterizou-se inicialmente pela exploração do conteúdo editorial a partir do processo de compreensão

indutiva, de onde emergiram as unidades de texto implícitas e explícitas identificadas. No processo dedutivo, observou-se que o aluno trabalhou em relação ao significado das frações como parte-todo e como índice comparativo entre duas quantidades ou conjuntos de unidades. Da mesma forma, destacou-se a importância de entender que as frações são razões que permitem demonstrar a comparação.

Palavras-chave: compreensão textual, operações fundamentais, dedutivo, indutivo.

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Introduction

One of the challenges of mathematics education is to develop and strengthen students' problem-solving skills in that discipline. Lee et al. (1996) have pointed out that success in achieving this goal is closely related to text comprehension and the solver's prior knowledge. More specifically, Jupri and Drijvers (2016) have reported on the difficulties presented in the students' written work, which are related to the understanding of words, phrases or sentences, as well as the formulation of mathematical models such as equations, schematics or diagrams. Krawitz et al. (2022) consider reading comprehension as an essential part in solving modeling problems, so improving it would result in developing modeling competence.

Indeed, despite the fact that students have the experience of reading various texts during their schooling, this does not guarantee that they understand those that are presented in later stages of their academic training. As a result, research has emerged to understand the decisive factors in the comprehension process of scientific texts in general and mathematical texts in particular (Carpenter & Lehrer, 1999; Murray et al., 1998).

Among the contributions to this topic, the evolution of epistemological obstacles to the precision of the object of understanding, the personal valuation of the object itself and the cognitive abilities of the subject stand out (Godino, 2000; Sierpinska, 1994); the methods focused on the construction of comprehension profiles (Pirie and Kieren, 1994); the semantic and structural analysis of the mathematical content proposed by Niemi (1996); supporting students' reading comprehension and its impact on enjoyment and performance in modeling problems (Krawitz et al., 2017); as well as the recognition of links between acts of flexibility as evidence of growth in mathematical understanding (Warner et al., 2003).

Understanding in mathematics is identified during classroom interaction, since it is there where a dialogue is established between multiple discourses, in which mathematical language is used (Barwell, 2015), particularly its vocabulary, syntax, and special abbreviations. (Simpson and Cole, 2014). The subjects that interact put into action their linguistic and communicative abilities to obtain information (Walkington et al., 2019), as well as their abilities to identify mathematical language words. (van Jaarsveld, 2016).

In mathematics classroom spaces, efforts made by researchers to assist in textual comprehension have been identified. For example, Graffigna et al. (2008) and Goñi (2009) recommend promoting comprehension in mathematics through reading, as well as taking actions to build meanings of the content being studied.

In this sense, the research carried out by Meaney and Flett (2006) shows a discrepancy between the objectives established in the textbooks and the drawing skills of the students. This is because the academic works focus on the completeness of the exercises, which limits the use of students' drawing skills as a tool to transform text into representations related to mathematical content.

On the other hand, works such as that of Tambychik and Mohd (2010) have focused on analyzing the difficulties that students face in distinguishing and analyzing the useful information of a problem, organizing a feasible solution, and using resolution strategies.

Durán and Bolaños (2016) identified the obstacles that basic level students face when working on problematic situations in mathematics. These drawbacks include the lack of identification of the main idea of the text, the execution of the analysis, synthesis and anticipation process required to process the information, and the lack of identification of the problem data to be interpreted. Along the same lines, Jiménez-Taracido et al. (2016) refer to the low awareness of the real level of comprehension of some readers, which demonstrates the lack of mastery of both evaluation strategies and regulation of their own comprehension.

For their part, Mato et al. (2017) report advances in the development of comprehension in mathematics with sixth grade students of primary education from ten centers in a municipality in Galicia (Spain), related to the regulation of learning and the recognition of their thought processes. However, they mention that the level of textual comprehension is still basic due to the low certainty of the operations and the explanations issued.

In the study by Miñano and Castejón (2011), the role of didactic strategies in student academic performance is highlighted, as well as in verbal aptitude, mathematical aptitude and mathematics grades. Likewise, Madero and Gómez (2013) mention the role of strategies to understand a text of mathematical problems, including the control of the strategies that the reader uses to construct a meaningful interpretation of the text during reading, and positive beliefs about themselves and reading, which act as precursors to thought and understanding.

According to Österholm (2005), works in mathematics education related to problem-solving reading include texts that are not necessarily statements of a problem, but may be the description of an assigned task. Therefore, problem solving regains importance in teaching and learning processes, since it acts as a trigger for cognitive activity due to the interaction that unfolds during its resolution (Osses and Jaramillo, 2008).

Having explained the above, this study investigates the textual comprehension processes of a high school student in Mexico when addressing mathematical problems with text. The theoretical position formulated by Duval (2004) bases the components of the textual comprehension process, particularly the segmentation and recontextualization operations, and the use of non-discursive representations for the understanding of textual organization and reconstruction.

The objective of the research was to analyze the processes of textual comprehension in mathematical problems with text by a high school student, for which the following research question was raised: how to characterize the segmentation and recontextualization operations in a high school student for the comprehension of texts in mathematical problems that involve text?

Theoretical elements

Text comprehension has been approached from various theories, in which the role of the reader in the construction of models to analyze the text stands out. In the previous section, the importance of these processes in solving mathematical problems has been exposed, in which elements such as the problem outline, strategies, objectives, and linguistic content, among others, are identified. In addition, the knowledge possessed by the reader and the information provided by the text are recognized as central variables in textual comprehension. In this scenario, complex and permanent cognitive abilities are

developed, self-reflection is allowed, and the choice of various strategies is made possible to understand a text effectively and with multiple objectives (Cáceres et al., 2018).

In this research, the theoretical position exposed by Duval (1999) is resumed, whose focus is oriented to the study of scientific texts. In it, two essential variables in textual comprehension are identified: the role that the reader acquires and the editorial characteristics of the texts. Duval (1999) points out that the analysis of text comprehension is only carried out through the interaction of these variables. When students are faced with problem situations with familiar information in a text, the mathematical formulation will be explicit to them. However, their performance will be limited when they have to solve problems in unfamiliar contexts and their knowledge base is restricted. In these cases, their interpretations of the textual information will be limited and the treatments to make explicit patterns or mathematical models, which may or may not be present in the problem, may be superficial.

Thus, Duval (1999) points out that for specialized texts mathematical treatments are required and that it is unavoidable to analyze the cognitive content of the statement, that is, the mathematical content implicitly and/or explicitly present in the text. This leads to carrying out two types of processes to promote textual comprehension. The deductive process demands two cognitive operations: the segmentation of the text into units and the cognitive recontextualization of the segmented units. When the situations are not familiar, an inductive comprehension process is required that also requires two operations: a functional segmentation oriented to the semantic analysis of the textual content in its entirety and a redactional recontextualization that is built from the different semantic relationships of the discriminated units. In both processes, the way these operations are carried out depends, on one hand, on the editorial type organization and the explicitation of the cognitive content and, on the other, on the knowledge base that the reader has about the cognitive content of the text.

The process of understanding a text begins with its decomposition into textual units of information (UTI). These UTIs are parts of the discourse whose function is to form a global organization of the text through relationships between its parts, where each one fulfills a function to configure the whole (Barletta and Chamorro, 2010). According to these authors, the UTI provide marks about the cohesion of the text, evidence a relationship with the reader's prior knowledge and other texts, which facilitates the reconstruction of meanings.

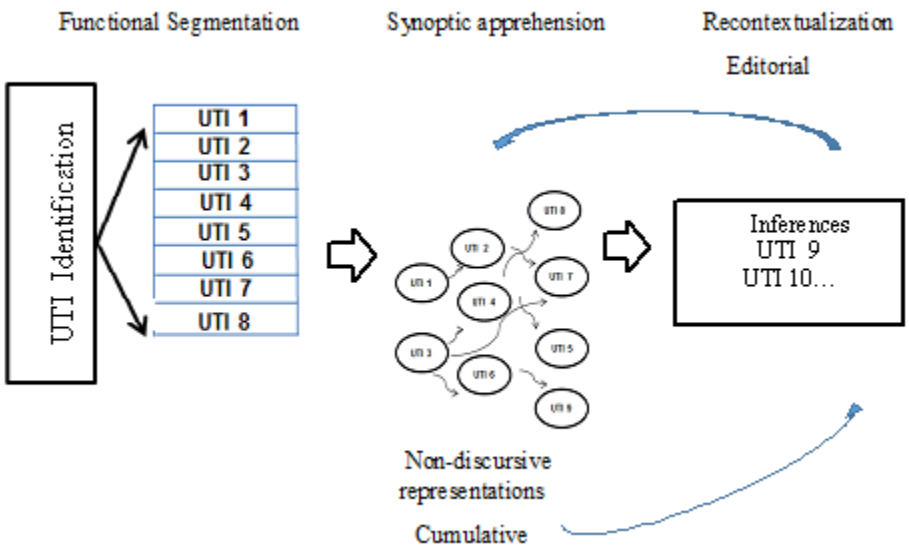
UTIs emerge from two types of segmentation: functional segmentation in the inductive process and cognitive segmentation in the deductive process. In both, a recontextualization operation is identified, which can be editorial for the first or cognitive for the second (Duval, 1999).

Functional segmentation focuses on editorial organization. The UTIs retain their place in the text and are identified based on main or independent propositions, which designate objects (proper nouns, nouns, adjectives, verbs) and complementary propositions what is said or can be said about the objects they appoint.

For its part, editorial recontextualization is one that makes explicit the existing relationships between the units discriminated against the functional segmentation. The UTIs discriminated by this type of segmentation are essentially the main propositions (designation of objects) and the complementary propositions (what is said about the object and that takes a certain value in the cognitive universe), whose relationships are varied: synonyms, oppositions, dependency or modeling (Duval, 1999, 2004). The set of these relationships constitutes the editorial organization of the text. To understand this organization, it is necessary to study the segmentation of the text, as well as the relationships established between the UTIs, regardless of their place in the text. This means that an object identified in a text can be enriched through the accumulation of information, which gives rise to a successive apprehension of the discriminated UTIs. When the UTIs find their place in the semantic network, the synoptic apprehension of said units has been realized (see Figure 1).

The means to carry out the objectification of segmentation and recontextualization operations are non-discursive representations, since they allow reconstructing and controlling the way in which each sentence is integrated into the text, which contributes to the synoptic apprehension of the textual organization that requires a multitude of successive apprehensions (Duval, 2004).

Figure 1. Textual comprehension in an inductive process



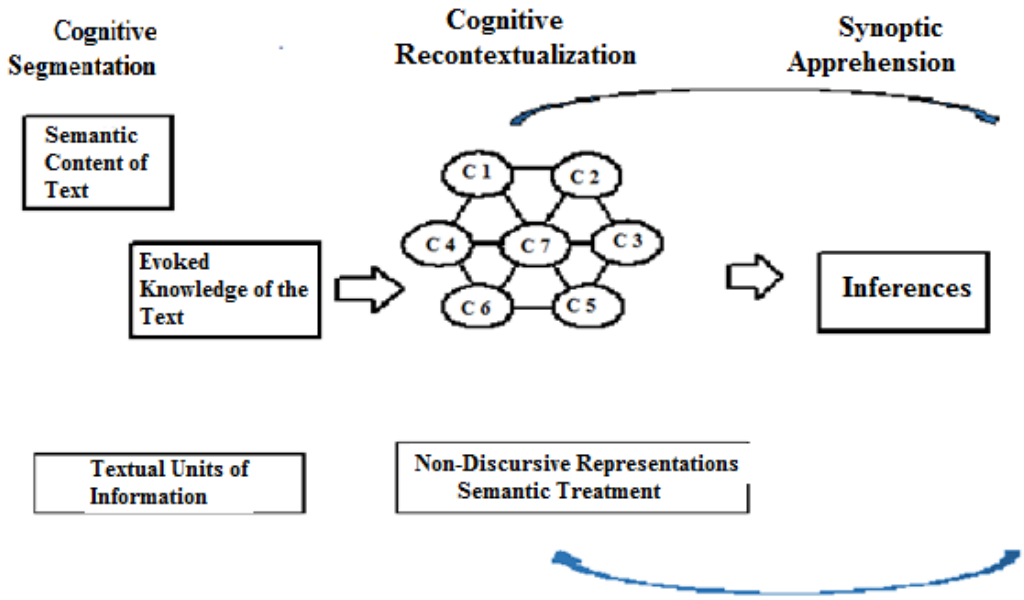
Note: UTI = Textual unit of information; main propositions, complementary propositions.

Adapted from Duval (1999)

The editorial recontextualization has the function of establishing the semantic relationships between the different thematic units identified in the text. This means that during the recontextualization process, the same object can be transformed or enriched by the accumulation of new information. Recontextualization is systematic and independent of the cognitive content, so its understanding requires the reader to have the ability to build and integrate the ideas present in the text (Duval, 2004; Guzmán et al., 2017).

Cognitive segmentation makes it possible to identify UTIs through questions focused on the conceptual schemes implicit in the text. The answers to these questions integrate the non-discursive representations of the situation presented in the text. Segmentation is selective and does not depend on the organization of the text. UTIs can be mathematical concepts, propositions, ideas, among others. Therefore, cognitive segmentation tends to eliminate explicit information units in the text that do not answer any of the questions focused on the conceptual schemata. Non-discursive representations emerge from systematic treatments —as shown in figure 2— and allow the reader to understand the text in a synoptic way, which gives rise to multiple inferences about the subject matter in the text. These inferences are necessary to fully understand the text, that is, to "read between the lines" (Cassany, 2019; Duval, 2004), and with this, discover elements not present in the text.

Figure 2. Textual comprehension in a deductive process



Note: UTI = Concepts, ideas, etc. Adapted from Duval (1999)

Cognitive recontextualization focuses on the production of inferences that are replaced by new inferences from modifications in the previous ones, based on mathematical treatments applied systematically and by substitution. When the UTI, identified through cognitive segmentation, takes the right place in the set of knowledge or ideas, deductive comprehension is carried out, which is independent of the editorial organization. In this sense, the variations that the wording of the text could present do not affect cognitive recontextualization because there are representations centered on the cognitive content of the statement, which is invariant in the face of any wording variation. In this way, the pertinent information for mathematical treatments is identified and the interpretation of the result is achieved in the situation analyzed (Duval, 1999, 2004).

From an intuitive perspective, it is possible to infer that the comprehension that depends on the inductive process focuses only on the explicitation of the text, while the comprehension that focuses on the deductive process focuses on the mathematical content of the phenomenon or problem presented. Finally, the comprehension that obeys the interaction of the inductive and deductive processes, and that induces to modify the wording, is called evolutionary comprehension, according to Duval (2004).

In the organization of the texts, it is also possible to identify non-discursive representations that constitute spaces for the apprehension of the discriminated units through the different forms of segmentation. The successive apprehensions depend on the recontextualization operation, to build the synoptic apprehension where the identified units find their place (Duval, 2004). In order to understand a passage of text, it is necessary to use the non-discursive representation as an intermediate to carry out treatments focused on the cognitive content or on the editorial organization.

The identification of the UTIs in the texts of mathematical problems is carried out through mathematical treatments and/or semantic relationships, depending on the type of segmentation used in the construction of inferences necessary for the understanding of the text. Therefore, it is necessary to use the non-discursive representations during the recontextualization operation. The non-discursive representations facilitate the global and successive synoptic apprehension of the UTIs discriminated and organized in a linear way. In this way, they are the basis for the construction of inferences in the comprehension of the text and for the detection of possible omissions or ambiguities during reading. According to Duval (2004), exploring non-discursive representations is fundamental to understanding texts.

Methods and procedures

Given the complexity of text comprehension processes in the mathematics classroom and the need to have a methodology that accounts for them, it was considered pertinent to characterize the segmentation and recontextualization processes through a case study that would allow analyzing comprehension and interpretation of the phenomenon from the reality of the person, and to seek the necessary rigor in the investigations that use it (Gil and Yamauchi, 2014). The case, described in the third phase of the investigation, was selected for presenting commitment and availability to explore activities that involved the comprehension of texts, and for his willingness to participate in a semi-structured interview.

The assumed vision of textual comprehension requires the analysis and interpretation of the segmentation operations carried out by the student to identify the UTIs, which allowed him to discern the explicit and/or implicit content of the text, either at the editorial or cognitive level. To do this, three phases were followed:

First phase: design of the problems with text

Eighteen text problems were analyzed and discussed during one semester of school classes, which included mathematical and non-mathematical contexts. The content of the problems was explored to carry out a functional and/or cognitive segmentation that would allow UTIs to be recognized. It is important to mention that the notion of text may or may not be familiar to an interlocutor, since it represents the written production of a discourse, or the object of a school practice and, therefore, it is a difficult notion to define.

In this sense, in the problems with text considered in this research, the following were identified as structural elements: an explicit context, not necessarily mathematical, whose formulation presents one or several solution alternatives according to the combination of stages to calculate intermediate unknowns and the possible creation of new problematic situations. For the classification of the problems, the structural elements were used: context, formulation, solutions and method, of the Borasi model (1986). According to the author, this type of problem presents an apparent content and identifies the necessary information for its solution, but the use of an algorithm, formula or calculation is not evident, and it has a real hypothetical situation, which is the one that somehow promotes the use of a method.

Second phase: data collection instruments

For the analysis of the textual comprehension processes of the student subject of analysis in the present investigation, worksheets were designed for the researchers-teachers and individual reports of the student's work were collected. It should be noted that the researchers-teachers worked with the student during the experience.

As shown in Table 1, the worksheet was used by all the researchers respecting the set of questions presented. The intention was to guide the student's reflection on the content offered by the text, as well as on the segmentation process.

Table 1. Worksheet for student identification of UTI´s

Questions	Segmentation operation	Description
1.- How many statements does the text present?	Functional segmentation	Segmentation in units preserving its place in the text.
2.- ¿Do you identify in the text words whose meaning you do not know? Specific	Functional segmentation	The meaning of the words in the text is recognized.
3.- Mention the relevant information to solve the problem. Justify your answer.	Functional segmentation	Words are identified that constitute the objects of propositions, which are usually nouns, pronouns, or adjectives. The verb is a key element that allows to identify the extension or limits of the proposition.
4.- ¿Is the text clear to you?, Why?	Functional segmentation	Complementary propositions are identified, things that are mentioned about objects whose production takes on a logical, epistemic or social value.
5.- ¿Mention additional information to solve the problem. Justify your answer.	Functional segmentation	Propositions in the text that are not significant to the reader are identified.
6.- ¿Do you identify information in the text that is related to each other? Explains	Functional segmentation	A semantic and connective network are identified. This refers to the identified semantic networks, in the order presented in the text, as well as the argumentative connectors.
7.- Describe with your words the situation that arises in the problem.	Functional segmentation	Editorial explicit information is identified.
8.- ¿What is the main idea in the situation? Justify your answer.	Cognitive segmentation	The idea that evokes or deals with the text is identified.
9.- ¿Do you identify information in the text that you could relate to previously learned concepts? Justify your answer.	Cognitive segmentation	Prior knowledge emerges, related to the situation, objects, or questions that the text evokes, regardless of those that the wording of the text makes explicit.
10.- ¿Do you consider that information is needed?, Why? And what would it be?	Cognitive segmentation	The answers delimit, a UTI that must be searched in the text.
11.- ¿Is there information in the text that does not agree with what you know? Explains.	Cognitive segmentation	Information not related to the ideas set forth in the text is identified.

Source: Authors, elaboration based on Duval (1999)

Third phase: selection of Bobby's case

Bobby is a 15-year-old student who was enrolled in an algebra course at a high school in Mexico. During the course, which lasted 18 weeks, he actively and motivatedly participated in all the activities proposed by the research-teachers. In the classroom, the researchers created a collaborative text problem-solving environment, in which Bobby participated with his classmates. At the beginning of each session of the course, the problems with text were raised, the students worked individually with the textual content and made individual reports. Bobby stood out for actively participating in various discussions, arguing about the positions expressed in individual reports, seeking consensus based on the interpretations put forward, and materializing discursive representations. In addition, he was in charge of organizing the work of the group of students with whom he interacted and preparing written reports.

The researchers selected Bobby and his work group (Lulú and Rosy) to carry out two work sessions after school hours. During the two sessions, the students carried out the segmentation and/or recontextualization operations. At the conclusion of the activities, Bobby participated in a one-on-one session. In the present investigation the process carried out by the student is presented.

Indicators for the analysis of Bobby's work

The exploration of the data obtained in the third phase provides evidence of the segmentation operations to identify the UTIs or recontextualization and of the construction of the non-discursive representations that were generated during the comprehension process. To carry out the analysis, three categories are proposed. The inductive process was called category I to signify the contents (see Table 2). The main interpretative referents were the operations of functional segmentation or editorial recontextualization. These allowed the recognition of grammatical phrases and propositions, semantic relationships, argumentative or temporal connectors, and possible inferences.

Table 2. Category I: Inductive process

Indicators	Interpretive Referents
I.1	Verbs, propositions.
I.2	Argumentative connectors.
I.3	Cumulative non-discursive representations.
I.4	Inferences.

Source: Authors, elaboration based on Duval (1999)

Category II corresponds to the deductive process, which includes cognitive segmentation operations to identify UTIs and cognitive recontextualization, as well as the construction of non-discursive representations that refer to knowledge, relationships with other knowledge, and the possibility of inferences. The indicators show the knowledge base that the reader possesses, separated from functional segmentation and editorial organization (see Table 3).

Table 3. Category II: Deductive process

Indicators	Cognitive segmentation and cognitive recontextualization
II.1	Non-discursive representations (systematic treatment).
II.2	Constituent knowledge.
II.3	Inferences.

Source: Authors, elaboration based on Duval (1999)

Category III includes indicators that allow understanding to be identified based on the interaction of the inductive and deductive process, which leads to the modification of the wording or of the situations emitted in the text. This is recognized as an evolutionary understanding (see Table 4).

Table 4. Category III: Evolutionary process

Indicators	Inductive and deductive process
III.1	Modification of the wording of the text.
III.2	Modification of the phenomenon and situation of the problem.

Source: Authors, elaboration based on Duval (1999)

With the proposed categories, the analysis of textual comprehension in problems with text in Bobby's case was carried out. It is worth mentioning that the indicators characterize categories I and II, while category III establishes the interaction of the inductive and deductive processes, which is part of the editorial organization and the activation of the interpretative scheme product of cognitive recontextualization. These categories place Bobby's work in category III, as the analysis will show.

Analysis and results

To support the credibility and relevance of the results, a triangulation of information was carried out from different sources: a) written report (individual), b) worksheet, c) video interviews and d) observations of the researchers. These sources allowed us to identify the inductive and/or deductive processes in textual comprehension.

The problem with applied text during the final activity (phase three) has a coherent text in which each line is linked to at least one other line through argumentative and/or temporal connectives. The problem presented in this work is a representative example of situations related to the meaning of the fraction as part-whole (Escolano and Gairín, 2005). The fraction is conceived as the relationship between two specific quantities, where the quotient in the fraction is the operation of dividing a natural number by another that is not null; the measurement has its origin in measuring quantities of magnitudes that, being measurable, do not correspond to an integer multiple of the measurement unit; and the reason refers to the meaning of the fraction as a comparative index between two quantities, since it is part of the proper nature of the fraction.

The problem with the text of the concert: Due to problems with the local authorities, the stage of a rock concert has to be changed (1). It should condition in less than 8 hours (2). One company can install the seats in 12 hours and charges $20,000, another takes 18 hours and charges $15,000 to do the same job (3).

1.- Will the concert be possible if the 2 companies are hired?

2.- In what terms should the contract be established so that the organizers pay as little as possible? (Delgado *et al.* 2001, p. 45).

The text explains the situation of a company (statement 1), which can be considered as a statement. Statement 2 expresses an epistemic value of certainty, while statement 3 establishes the conditions and costs to perform the work. With these characteristics, the text has explicit content and each line is linked, at least, with another line of the text (Borasi, 1986).

To analyze the text comprehension processes experienced by Bobby, it begins with a description of the work done by the student. A table is prepared to analyze how Bobby describes the situation, another to analyze how he develops the functional segmentation and/or cognitive segmentation of the text in UTI and one more to recognize how he organizes the UTI. After each table, an analysis of the information they contain is written, and based on these tables, a network of Bobby's work relationships is built, based on the theoretical elements included in the corresponding section. From the network of relationships, an analysis of the inductive and deductive processes is carried out.

Through the interviews and observations carried out by the researchers-teachers, the segmentation and recontextualization operations of the units segmented by the student were confirmed or refuted. Emphasis was placed on the editorial and/or cognitive

recontextualization operation to make explicit the existing relationships between the UTIs that emerged from the editorial organization or the evoked knowledge. This allowed recognition of the inferences made by Bobby.

Inductive process

The investigators asked Bobby to explain the situation and take a stand. He identified the conditions and requirements to carry out the concert under the guidelines established by the companies.

> *01 Bobby: Some seats have to be moved in less than 8 hours to be able to hold a concert and there are 2 companies that can take care of the job, but at different times and costs.*

Analysis and discussion of the segmentation: For the analysis of Bobby's work, Table 5 was prepared, consisting of one line per statement. In the first column the statement number is written, in the second the connectives that unite the propositions, in the third the main propositions where the reference objects are exposed, in the fourth the verbs that express a propositional attitude are mentioned —that is to say , have a descriptive function in the expressed propositions and these verbs are generally presented in the infinitive—, and the complementary propositions are located in the last column. To make the propositions explicit, referents are placed with the "prime" notation.

Table 5. Situation exposure (segmentation description)

Statement number	Connective	Main propositions	Verbs	Complementary proposals
1		A few seats	They have to move	**1´** in less than 8 hours
2	To be able to	A concert	Carry out	
3		There are two companies	Can Take care	**2´** of work
	But			**3´** at different times and costs

Note: Own elaboration with information from Duval (1999)

In Bobby's work performing the concert is identified as thesis. The argument to refute or accept the thesis is expressed in 1'. Next, he establishes the conditions in 3' and

argues in statement 3 the options to carry out the concert. When questioning Bobby the fundamental idea of the situation, he mentions:

> *0.2 Bobby: Getting seats on stage in less than 8 hours, because that's where the problem comes from.*

In this way, it is recognized that the question established in the text —In what terms should the contract be established so that the organizers pay as little as possible?—, acquires a secondary status.

During the functional segmentation operation, Bobby identified the UTIs. To explain how he performed the functional segmentation and/or cognitive segmentation of the text in UTI, Table 6 was designed with five columns: in the first one are the argumentative connectives (in capital letters and bold) that join the propositions, in the next two are the they identify the main propositions made up of the verbs that express a propositional attitude (in italics and bold), and in the last columns the propositions that complement.

Table 6. Text segmentation

Argumentative connectives	Main propositions		Complementary proposals	
1 BUT	The problem	Specifies	*1´* How long does each company take?	How much you charge for that time?
2	The company 2	*Can't do*	*2´* All the work	SO the participation of both companies is necessary
3	The second company	Is	3´ the cheapest	
4 BUT		*Must*	*4´* work as long as possible	

Note: Segmentation of text into text units of information. Argumentative connector in capital and bold. Propositional attitude in italics and bold.

The UTIs recognized during the segmentation of the editorial type based on the corresponding indicators are three: (i) the main propositions that designate the object, whose form of expression are proper names, nouns, qualifying adjectives or indefinite pronouns; (ii) complementary propositions, that is, what is said about the object, where the verb expresses a propositional attitude and is the copula between a subject (noun) and a predicate (attribute); and (iii) the argumentative connectors that have the function of justifying the different approaches to give coherence and meaning. Within this framework,

it was identified that Bobby strictly adhered to what was mentioned in the situation and established relationships to determine the procedure that led him to propose the ideal solution scenario.

Analysis and discussion of the editorial organization: Recognizing the relationships established between the ICUs opened an opportunity to rethink the editorial organization and consider possible inferences. This analysis was carried out using the characterization proposed by Duval (2004), which allowed the elaboration of an interpretative scheme of what Bobby exposed (see Figure 3).

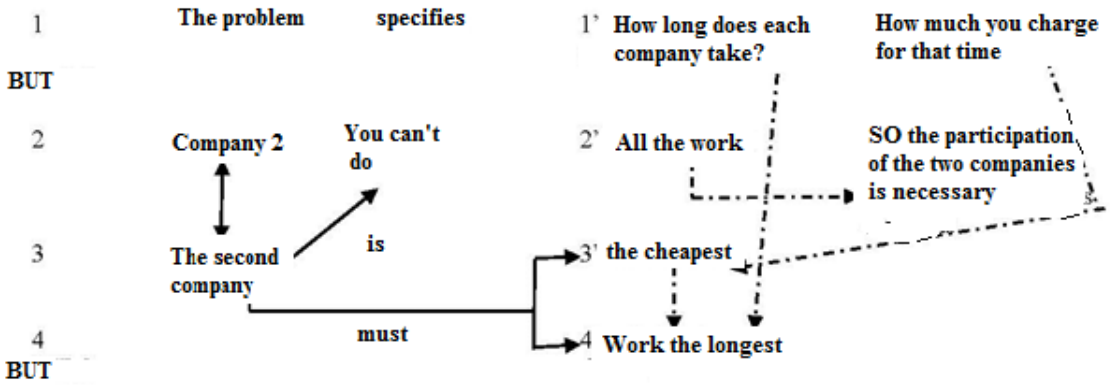
Figure 3. Relationships used to rethink editorial organization

Relationships between UTIs	Language: Syntax and Lexico	Inferences to justify editorial coherence
Synonyms	Replace the term with a variation ⟷	Replace the complement in the proposal with another of equal meaning ⟷
Dependence	Noun that generates dependency →	Propositions that remain related based on their use →

Source: Adapted from Duval (1999)

The relations between the UTIs in the editorial organization show the interpretation and the operation of editorial recontextualization used by Bobby. In this sense, the first component is the synonymy assigned by the syntax (word) and the wording coherence (propositions). The second is the repetition of a term and the proposition, and finally, the dependency established by nouns that denote adherence. As for propositions, they retain connections. The networks built with the UTIs make up the recontextualization of the editorial content (see Figure 4).

Figure 4. Editorial recontextualization. Construction of the network of relationships established in the text



Note: Relationships are established between UTIs

The relationships established by Bobby present two important parts: (1) the way of organizing the conditions for each company based on what is required by the organizers of the event, and (2) the thesis on the longer work time by the company 2 due to at the costs presented. The text specifies the participation of two companies with a specific time and at the lowest possible cost (proposition 1'). The argument for company 2 to collaborate eight hours is given in 3' and 4', the down arrow represents the relationship between them; Hence, proposition 2' expresses the need for the participation of companies, specifically in 3', compliance with the organizers' requirement to pay the lowest cost, and the conditions of company 2 are detailed.

Requests to companies are reorganized based on what is required to carry out the event. This is how Bobby makes the decision at 3' and 4' to establish that company 2 must work 8 hours because it is the cheapest, and that company 1 must also intervene to hold the concert on time. With this, he establishes semantic relationships between expressions and uses argumentative or temporal connectors (redactional recontextualization operation).

On one hand, the student makes the decision, in 3 'and 4', to consider that company 2 must work 8 hours because it is the most economical, with which he establishes the first inference. On the other hand, he argues the limitation of company 2 to complete the work and, at the same time, expresses the need for the participation of company 1, second inference. The aforementioned exposes the synoptic apprehension of the editorial organization, in which the units identified by functional segmentation take their place, and

the conditions are established to mention initial inferences from a non-explicit global approach in the text, as mentioned McKoon y Ratcliff (1992).

Deductive process

After explaining the problem, Bobby moves away from the structure of the text to generate propositions that conform to the modifiable order inherent in understanding and to reorganize the segmented units into a body of knowledge relevant to the topic. This makes it possible to generate inferences.

To analyze the results of Bobby's deductive process, the non-discursive representations that the student used were identified. Then, the knowledge related to the topic of the discipline addressed was examined and propositions that were analyzed through cognitive recontextualization were recognized. This allowed segmented units to be relocated without considering the problem text. The resulting platform consisted of a set of knowledge that made it possible to generate various inferences and discover implicit information in the situation posed. The analysis of the identified inferences was based on the use of a third statement to move from a proposition to a conclusion. Finally, a scheme was built to systematically present the propositions.

The non-discursive representations used were numerical and algebraic, in which conceptual and operational treatments were recognized. These representations were used to understand the cognitive organization of the student. Implicit information treatments in the cognitive content of the situation were also explored, which led to the construction of inferences that helped to solve the problem.

Constituent knowledge requires the application of a rule, either implicit or explicit. In this case, proportionality was used to pose and solve the problem. The ratios are considered as the relationship between two magnitudes, whether they are of the same nature or different (Hart, 1988).

Bobby's dissertation consists of two sections: the presentation of the premise about the willingness of companies to change the scenery at a rock concert, and the argument that evidences the decision making. In this way, the participation of the two companies is determined according to time and cost. Bobby sets the condition that Company 2 must work 8 hours due to costs, which satisfies the organizers' requirement to pay as little as possible. This assertion acquires truth value and is justified with the following statement:

03. Bobby: The second company is the cheapest, but you have to work the longest.

The student began the cognitive recontextualization by relocating the segmented units in a body of knowledge, which led to delinearization.

> *I.- We do not know the number of seats, so in 8 hours "x" seats must be moved*
> *II.- but it would only end* $\frac{8}{18}x$ *in the 8 hours that you have at most to finish the job* $\frac{8}{18}x$ *It represents* $\frac{8}{18}$ *of the seats and*
> *III.- I obtained it from the hours that company 2 takes to finish the complete job, which would be* $\frac{18}{18}$
> *IV.- but since they only have 8 hours to finish, they will only work for 8 of the 18 hours, so they would advance* $\frac{8}{18}$ *from work*
> *V.- Of the others* $\frac{10}{18}x$ *remaining, company 1 will have to commission. Company 1 would finish the work in 12 hours, which would be equal to* $\frac{12}{12}x$,
> *VI.- but since company 2 will take care of* $\frac{8}{18}x$
> *VII.- then: the subtraction is made*
> $\frac{12}{12} - \frac{8}{18} = \frac{\frac{20}{3}}{12}$ *so company 1 will be in charge of* $\frac{\frac{20}{3}}{12}x$

Analysis and discussion of propositions. Proposition I shows the explanation and the follow-up in the algebraic representation. It exposes a semantically neutral operation based on the variable "x", which represents the number of seats to be assigned, and shows the magnitude and direct proportionality as a first intuitive approach. In addition, it highlights the time established by company 2 and the requirement of the businessmen to complete the work in 8 hours and guarantee the performance of the concert.

Propositions II, III, IV, V and VI refer to the working time condition (8 hours) which is the fundamental premise, which delimits the work that corresponds to company 1 and is represented as $\frac{10}{18}x$. Finally, the operation is executed in proposition VII and concludes with the simultaneous time that both companies must work (6.6 hours) to stipulate that only company 2 ends with 8 hours. In accordance with this approach, the determined costs are respectively company 2 the amount of $6666.6 and company 1 $11 111.11.

The recontextualization made possible the successive apprehension of a set of knowledge organized in a systematic way to conclude with proposition VIII:

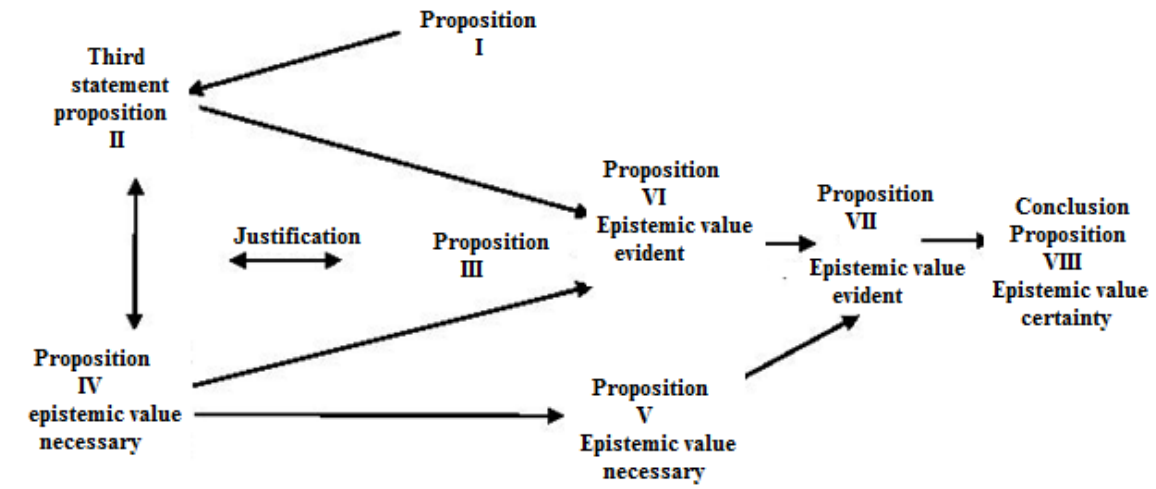
> *VIII: Company 1 would work 6.6 hours for $11,111.11 → Result of 6.6 hours for the cost/hour.*
> *Company 2 would work 8 hours for $6666.6 → Result of 8 hours for the cost/hour. Total to pay: $17,777.77"*

In the student's work, it is recognized that he resorts to the part-whole meaning when he admits the fraction as the relationship between two specific quantities.: $\frac{8}{18}$ *of the seats* y $\frac{10}{18}$ *remaining, highlighting a whole and a part.* In addition, the meaning of the fraction as a comparative index between two quantities or sets of units is present, the fraction is a ratio where the comparison between two values is evidenced, and the order in which the magnitudes are presented is also recognized as necessary. These results agree with Lamon (2007) regarding the proportional reasoning in the discrimination of the constant ratio between components of the same measurement space and the discrimination of the scalar connection between different measurements. Note the ability to explore a multiplicative relationship between two quantities.

Regarding the inferences inherent to understanding, they are characterized by the direct passage from one proposition to another to issue a statement or statement related to a particular context imposed by a concept or principle (León, 2001). The recontextualization operation is completely based on the exploration of representations foreign to the text, integrated by a set of knowledge, which causes the successive identification of various inferences to eliminate possible ambiguities or discover implicit or missing information and situate itself in the interpretive scheme of the solver. .

Knowledge is recognized through systematically organized propositions, and development results in a synoptic apprehension of cognitive organization in which units find their place and relationships with other more distant units are perceived. Therefore, the inferences identified depend on the semantic organization. However, the passage from a proposition to a conclusion was made with the support of a third statement that is related to statements of a semantic type. The inferences are identified with the help of figure 5.

Figure 5. Analysis of identified inferences



Note: Presentation of systematically organized proposals

Analysis and discussion of inferences. The sequence from II to IV is justified with III, since it argues the fulfillment of the 8 hours of work required by company 2. Sequences V and VI establish the conditions corresponding to company 1 and state the reliability of each proposition. The passage from V to VI gives rise to the inference VII, and finally the conclusion is stated in VIII, whose epistemic value is certainty. The epistemic component operates as an instrument to restructure and renew knowledge (Serrano, 2014). With this proposal, Bobby demonstrates that it is possible to carry out the concert if both companies are contracted and establishes the terms in which the contract must be established so that the organizers pay as little as possible. Proposition II allows us to establish the conclusion in VIII.

The text evokes for Bobby a known mathematical experience in relation to a concrete mathematical knowledge. This is for the student a scenario of reflection and cognitive imbalance known to him, which leads him to decide to address, elaborate and present answers, which makes significant use of this knowledge.

The evidence present in the analysis explains how, during the text comprehension learning experience in the mathematics classroom, segmentation and recontextualization operations are promoted while Bobby explores the textual content and externalizes the knowledge base that he possesses. This allows to identify the inductive, deductive or evolutionary processes during the construction of inferences.

Importantly, student performance is affected when he is confronted with unfamiliar situations, leading to a variety of interpretive events related to Bobby's understanding. An

example of this is the choice of companies before the use of mathematical knowledge, which generated interpretations that confront and take precedence over each other, although not always correctly.

Evolutionary process

Based on the above, it is concluded that the evolutionary comprehension carried out by Bobby is closely related to the segmentation and recontextualization operations of the segmented units, the latter being essential to achieve textual comprehension. In addition, the editorial and cognitive content of the text is explored, as well as the knowledge base that the reader possesses. To do this, Bobby performs two processes during the analysis: inductive and deductive, which allows him to consolidate various inferences that are essential for learning through reading in mathematics.

The exploration of the inductive and deductive processes shows the importance of strengthening developmental comprehension during the comprehension of texts in the learning of mathematics. Both processes contribute to exploring the two fundamental operations (functional segmentation and recontextualization) that allow the identification of editorial content, as well as the explicitation of cognitive content.

Discussion

In Bobby's work, two processes related to the textual comprehension of a mathematical problem with text were identified. The first was inductive, characterized by the identification of the thesis, the realization of the argument and the establishment of conditions for it to be fulfilled, initially granting a secondary status to one of the questions mentioned in the text. Based on this, Duval's (1999) theoretical framework provided conceptual tools to identify UTIs and recognize a redactional segmentation in student work. The UTIs are related through synonyms and dependencies (Duval, 2004) and make up a network, called redactional, which is built with the propositions to build the first inferences that will lead to a deductive process.

In the deductive process, Bobby moved away from this wording organization while building propositions based on prior knowledge related to the topic, such as the concepts of fractions as a comparative index between two quantities and proportional reasoning. He started with the proposal of second inferences that allowed him to make decisions to approach the solution of the problem posed.

Both processes distinguish Bobby's work from what another student could do, since it is related to his academic profile, strategies used, propositions used, prior knowledge, among others. These findings coincide with what was reported by Barletta and Chamorro (2010) who mention that a text produces a social activity in which a subject organizes and presents information in a way that enables him to relate his knowledge, respond to his expectations and interests.

Duval (2004) and Graffigna et al. (2008) expose the difficulties present in high school students in two dimensions, one operational, referring to the concepts involved, and another related to the cognitive structure, both of which have an impact on textual comprehension. Evidence of Bobby's work on the above dimensions is reported in this document. In the operation, the student analyzed the text and identified the UTIs and, in the second, he established inferences from mathematical treatments. As in the works by Duval and Graffigna, difficulties were identified in the operational dimension, which is related to the lexicon present in the statement, and in the cognitive dimension, they were limitations to establish the networks between the propositions present in the text. Jiménez-Taracido et al. (2016) also refers to the difficulties related to the lack of understanding of the text, that is, decoding the words shortening the reading process at the lexical level, a difficulty also present in what Bobby did.

A limitation of the present study is that the data is derived from a case analysis, which implies the need for additional research of a similar nature to that described by Duval (2004) in order to broaden the understanding of the impact of findings in a broader spectrum of the population. In addition, the importance of undertaking longitudinal research that focuses on the learning process is suggested in order to evaluate the impact and effectiveness of the proposed activities for students.

Final considerations

The present study examined Bobby's work during the comprehension of texts in mathematics, through the processes of segmentation and recontextualization in mathematical problems with texts. Bobby's performance in the investigation of the proposed situation is shown to be unique from the perspective of his academic profile, and demonstrates the functional and cognitive segmentation applied to analyze the processes of text comprehension. In addition, the student explored structural, semantic and

communicative aspects linked to the knowledge involved in the situation, which strengthened their synoptic apprehension of the content.

In general, the use of the worksheet benefits the student to reflect on the propositions presented in the text and carry out the editorial segmentation, which depends on the familiarity of the reader with the topic addressed in the situation and the explicitation of the cognition content in the text.

It is possible to infer that student performance in text comprehension is evolutionary and strengthens learning in mathematics. However, it is advisable to consider other conditions, such as problems with high information in mathematical content, contextualized problems, word problems, and the reader's prior knowledge to investigate the content.

Future lines of research

It is a priority to study the processes of text comprehension in the teaching-learning process to establish ideal scenarios in strengthening the inductive and deductive processes in learning through reading.

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