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Artículos científicos

Impacto de los conocimientos previos de álgebra y aritmética en el aprendizaje de funciones de cálculo diferencial

Impact of prior knowledge of Algebra and Arithmetic on learning differential calculus functions

Impacto do conhecimento prévio de álgebra e aritmética na aprendizagem de funções de cálculo diferencial

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Resumen

El cálculo diferencial es una de las áreas de las matemáticas que presenta mayor complejidad, de ahí que, en muchos casos, constituya un verdadero reto para los estudiantes. Una de las razones por las que esto ocurre se debe a que para abordar el cálculo diferencial es necesario tener conocimientos sólidos en áreas previas de la matemática, como lo son el álgebra y la aritmética. En este sentido, el objetivo de esta investigación es analizar el impacto de los conocimientos previos del álgebra y la aritmética en el aprendizaje de funciones en el cálculo diferencial. Para ello, se empleó un paradigma cuantitativo, por medio de un estudio no experimental y exploratorio, con una muestra de 27 estudiantes. Asimismo, se aplicó un análisis de correlación que permite concluir que la calificación de conocimientos previos está positivamente relacionada con la calificación de funciones; además, no se presentó una diferencia significativa en la calificación de funciones y el sexo, y tampoco la edad es un factor importante para predecir el desempeño en funciones.

Palabras clave: cálculo diferencial, conocimientos previos, funciones, matemática educativa.

Abstract

Differential calculus is one of the most complex areas of mathematics and, in many cases, represents a real challenge for students. One of the reasons for this is that, to approach differential calculus, it is necessary to have solid knowledge in previous areas of mathematics, such as algebra and arithmetic. In this sense, the objective of this research is to analyze the impact of prior knowledge of algebra and arithmetic on learning functions in differential calculus, under a quantitative paradigm, through a non-experimental and exploratory study with a sample of 27 students and a correlation analysis.

It is concluded that the score of prior knowledge is positively related to the score of functions, and there is no significant difference in the score of functions and gender. Age is also not an important factor in predicting performance in functions.

Keywords: differential calculus, prior knowledge, functions, mathematics education.

Resumo

O cálculo diferencial é uma das áreas mais complexas da matemática, razão pela qual, em muitos casos, constitui um verdadeiro desafio para os alunos. Uma das razões pelas quais isso ocorre é porque para abordar o cálculo diferencial é necessário ter conhecimentos sólidos em áreas anteriores da matemática, como álgebra e aritmética. Nesse sentido, o objetivo desta pesquisa é analisar o impacto do conhecimento prévio de álgebra e aritmética na aprendizagem de funções em cálculo diferencial. Para isso, utilizou-se um paradigma quantitativo, através de um estudo não experimental e exploratório, com uma amostra de 27 estudantes. Da mesma forma, foi aplicada uma análise de correlação que permite concluir que a classificação do conhecimento prévio está positivamente relacionada com a classificação das funções; Além disso, não houve diferença significativa nas classificações dos papéis e no género, e a idade não é um factor importante na previsão do desempenho dos papéis.

Palavras-chave: cálculo diferencial, conhecimentos prévios, funções, matemática educacional.

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Introduction

Differential calculus is a branch of mathematics that is responsible for the study of the properties and behavior of functions. This discipline is essential in many fields of science and engineering, including physics, economics, biology, and computer science, among others. However, learning differential calculus can be challenging for many students due to the complexity of its concepts and the need to develop advanced problem-solving skills.

One of the factors that can influence students' success in learning differential calculus is the strength of their prior knowledge of algebra and arithmetic, which is essential to understanding the properties of functions since students need to know how to solve equations, manipulate algebraic expressions, and understand numerical structure. Furthermore, the ability to apply these mathematical tools is critical to solving complex problems in differential calculus.

Since understanding functions is a crucial component of differential calculus, the relationship between prior knowledge of algebra and arithmetic and performance in learning functions in this discipline has been widely investigated. Existing literature suggests that a solid understanding of algebra and arithmetic concepts is a necessary condition for students to successfully address function concepts in differential calculus.

Therefore, the goal of this article is to examine in depth the impact of prior knowledge of algebra and arithmetic on learning functions in differential calculus. To achieve this objective, the existing literature on this relationship was reviewed, which allowed us to describe a method to examine it. Additionally, the implications of the results for the teaching of differential calculus are discussed and suggestions are offered to improve students' understanding of this fundamental field of mathematics.

Literature Review

Existing literature indicates that prior knowledge of algebra and arithmetic is vital for learning functions in differential calculus. In particular, students must be comfortable with algebraic manipulation of equations and expressions, as well as understanding numerical structure to successfully address the concepts of functions in differential calculus. In this regard, Pérez de Paz (2019) points out:

> knowledge is a principle of constructivist pedagogy that, based on cognitive theories, suggests that the subject is capable of developing his or her knowledge construction processes, in this way, the student upon entering

school, level or grade, already has a wealth of knowledge, which allows it to start a new learning process and defines, at the same time, the teaching process that will be developed by the teacher, therefore, the teaching-learning process does not start from scratch, nor is it considered that the student does not know (p. 4).

In other words, prior knowledge refers to the concepts, skills, attitudes, and experiences that a student possesses before learning a new topic or concept. These can be acquired through formal education, everyday experience, culture, and social interaction, among others. Students' ability to understand and apply concepts related to functions depends largely on their mastery of the fundamental concepts of arithmetic, so students must have a solid knowledge of basic mathematical operations such as addition, subtraction, multiplication, and division. In addition, they must be familiar with the concepts of fractions, exponents, roots, and logarithms, since these are essential for understanding functions.

Another important prior knowledge in learning functions in differential calculus is algebra. Students should therefore have strong skills in manipulating algebraic equations and expressions, including the ability to simplify complex algebraic expressions and solve linear and quadratic equations (Rojas Maldonado y Toscano Galeana, 2022). Without a solid understanding of algebra, students may have difficulty understanding the structure and properties of functions, which can lead to conceptual, procedural, and mathematical reasoning errors.

Scorzo 's study *et al* . (2009)on mathematical skills for differential calculus in engineering students points to the didactic importance of using *software* for the visualization and management of symbolic and numerical calculation, which frees the student from this task to emphasize the reasoning of concepts and deductions. That is, the purpose is to further promote conceptual, metacognitive, and heuristic skills.

For his part, Rodríguez Hernández *et al*. (2022)point out that working by competencies requires that the student understand learning as a multidirectional circuit that must be managed in a critical, ethical, creative, and sensitive manner to promote their comprehensive training. Similarly, Vrancken *et al*.(2006) mention that students may have prior knowledge and fail in the study of calculus. These authors explain that knowledge arises from ruptures of previous knowledge, so it is not a continuous process. In this sense, García Retana (2013) considers the following:

> The failure of students to learn calculus could be related to inadequate handling of mathematics due to a lack of knowledge of how to take advantage of the resources offered by algebra, the properties of numbers, equations and inequalities, geometry, and especially functions (p. 37).

However, López Recacha (2009) accurately states that if prior knowledge is non-existent, it must be replaced. That is, if prior knowledge is disorganized or erroneous, it is advisable to solve it before starting to learn the new content, but if prior knowledge is sufficient to address the new knowledge, it does not ensure that it is kept in mind at all times throughout the course. of the entire learning process.

This idea, in a study by Scorzo *et al* . (2009), where you assign primary importance to teaching the use of *software* to leave aside the use of numerical and algebraic calculations and focus on the understanding of concepts, it is suggested that the student transcribes the results that the *software* offers without reflecting. in the results that are expressed, which may become incoherent. In this work, it can be seen that these errors are related to the concepts, and not to the use of the *software*, which is similar to the results of this work.

Likewise, Vrancken *et al* . (2006)indicate that the difficulties related to the concept of function are the representation of graphs, the concept of function, and the difficulty of distinguishing between dependent variable and independent variable, which can be a persistent problem, which agrees with the results of the present work; However, Vrancken 's study *et al* . (2006) does not propose a methodology that is established and founded in this work.

In this order of ideas, García Retana (2013)affirms that the failure to learn calculus could be due to ignorance of algebra and the properties of numbers, a purpose that this work seeks to demonstrate and substantiate statistically.

Likewise, Noto *et al* . (2018)carry out a similar study and focus on analyzing the obstacles to learning the limit, since students demonstrate difficulties in factoring, multiplying factors, etc., aspects present in our work (regarding prior knowledge), and which are incorporated and characterized within the instrument they designed Rojas Maldonado y Toscano Galeana (2021).

Methodology

To examine the impact of prior knowledge in algebra and arithmetic on the learning of functions within the context of differential calculus, a convenience sample composed of 27 university students distributed in two groups (3 and 4) of the bachelor's degree was selected. in Biotechnology from the Michoacana University of San Nicolás de Hidalgo, corresponding to the second semester of the 2023-2023 cycle.

Before the start of the differential calculus course, an instrument designed to evaluate prior knowledge in understanding the concept of limit was administered (Rojas Maldonado and Toscano Galeana, 2021). Subsequently, after the face-to-face instruction phase on functions, the evaluation was carried out using a form in Google Forms.

This research, of a quantitative and exploratory nature, is part of a non-experimental study. The data collected covered prior knowledge scores in algebra and arithmetic, age, gender, group, and role qualification. These data were subjected to a descriptive statistical analysis in SPSS to calculate means, standard deviations, and relative frequencies for each variable. Additionally, a correlation analysis was used to examine the relationship between prior knowledge in algebra and arithmetic and performance scores.

Results

Table 1. Participant statistics

		Sex	Cluster	Age	Previous	Features
N	Valid	27	27	27	27	27
	Lost	0	0	0	0	0
Half			3,520	18,810	6,032	4,850
Median			4	19.00	6,430	4.00
Fashion			4	18	4.29a	2a
Desv. Deviation			.509	.921	1,576	2,381
Minimum			3	18	2.86	2
Maximum			4	twenty-one	9.29	10
a There are multiple modes. The smallest value is displayed.						

Source: self-made

Table 1 shows that 27 students participated throughout the study. The results of the statistical analysis show that the mean score of prior knowledge of algebra and arithmetic is 6.03, with a standard deviation of 1.57, where the minimum score was 2.86 and the maximum obtained was 9.29, while the mean score of functions was 4.85 with a standard deviation of 2.381, where the minimum score obtained was 2 and the maximum 10. Regarding sex, 51.9% of the sample are men and 48.1% women, while the average age was 19. years, with a standard deviation of 0.92, with 18 years being the minimum age and 21 the maximum. The sample consisted of 48.1% of students from Group 3 and 51.9% of students from Group 4.

Table 2. Age of participants

Age					
		Frequency	Percentage	Valid percentage	Accumulated percentage
Valid	18	12	44.4	44.4	44.4
	19	10	37.0	37.0	81.5
	twenty	3	11.1	11.1	92.6
	twenty-one	2	7.4	7.4	100.0
	Total	27	100.0	100.0	

Source: self-made

Table 2 reflects the percentage of grouping of the participants according to age, that is, 44.4% corresponds to the age of 18 years (the largest group in the sample) followed by 37% to the age of 19 years. There were only 2 students with the age of 21 years: a man from group 4 and a woman from group 3).

Table 3. Students who obtained a grade greater than or equal to 6 in both grades.

		Cluster	Sex	Age	Previous	Features
N	Valid	8	8	8	8	8
	Lost	0	0	0	0	0
Half		3.63		18.13	7,590	7.88
Median		4		18	7.5	8
Fashion		4		18	7.14a	8
Desv. Deviation		.518		.354	.850	1,246
Minimum		3		18	6.43	6
Maximum		4		19	9.29	10
a There are multiple modes. The smallest value is displayed.						

Source: self-made

Table 3 shows that only three students (37.5%) passed both exams in group 3, and five students (62.5%) in group 4. That is, in total, eight students passed, of which four were women and four men, seven at the age of 18 years and one with at the age of 19 years. The average prior knowledge score was 7.59 and the average feature score was 7.8, with very similar deviations of 0.85 and 1.2, respectively.

Table 4. Students who obtained a score greater than or equal to six in prior knowledge.

		Frequency	Percentage	Valid percentage	Accumulated percentage
Valid	6	4	28.6	28.6	28.6
	7	5	35.7	35.7	64.3
	8	4	28.6	28.6	92.9
	9	1	7.1	7.1	100.0
	Total	14	100.0	100.0	

Source: self-made

Table 4 shows the grades of 14 students who showed they had adequate prior knowledge: eight men and six women, who make up 57.1% and 42.9%, respectively, distributed equally in the two groups; nine of them were 18 years old, two were 19 years old, two were 20 years old and one was 21 years old.

Table 5. Students who obtained a grade greater than or equal to six in functions

		Frequency	Percentage	Valid percentage	Accumulated percentage
Valid	6	3	30.0	30.0	30.0
	7	2	20.0	20.0	50.0
	8	3	30.0	30.0	80.0
	9	1	10.0	10.0	90.0
	10	1	10.0	10.0	100.0
	Total	10	100.0	100.0	

Source: self-made

Table 5 shows the grades of 10 students (37%) who showed knew functions, six being men and four women, four in group 3 and six in group 4, eight aged 18 years, and two aged 19 years.

Table 6. Students who obtained a score less than 6 in prior knowledge, and a score greater than or equal to 6 in functions.

		Frequency	Percentage	Valid percentage	Accumulated percentage
Valid	4	1	50.0	50.0	50.0
	5	1	50.0	50.0	100.0
	Total	2	100.0	100.0	

Source: self-made

Table 6 shows that only two students did not demonstrate sufficient prior knowledge, with scores of four and five, belonging to the groups, aged 18 and 19 years.

Table 7. Correlations

		Previous	Features	Age
Previous	Pearson correlation	1	.423*	-.204
	Sig. (bilateral)		.028	.306
	N	27	27	27
Features	Pearson correlation	.423*	1	-.539**
	Sig. (bilateral)	.028		.004
	N	27	27	27
Age	Pearson correlation	-.204	-.539**	1
	Sig. (bilateral)	.306	.004	
	N	27	27	27
*The correlation is significant at the 0.05 level (two-sided).				
**The correlation is significant at the 0.01 level (two-sided).				

Source: self-made

Table 7 reveals a Pearson correlation of 0.423 between *prior knowledge* and *functions*, indicating a moderate positive correlation between these two variables. This suggests that there is a statistical relationship between having a higher level of prior knowledge and obtaining a higher grade in the function evaluation.

Furthermore, a weak negative correlation (-0.204) is observed between age and prior knowledge score, suggesting that younger students tend to have higher prior knowledge scores. However, age and function rating do not present a significant correlation, indicating that age is not a relevant factor in function rating. This shows that they are independent variables and should be considered individually in any further analysis.

Table 8 . Model Summary

Summary of model [b]								
					Change statistics			
Model	R	R squared	Adjusted R-squared	Standard error of the estimate	Change in R squared	Change in F	gl 1	gl 2
1	.423 to	.179	.146	2,200	.179	5,452	1	25

to. Predictors: (constant), priors

b. Dependent variable: functions

Source: self-made

Performing a simple linear regression analysis as the dependent variable, the rating of functions, and as the independent variable, the rating of prior knowledge, the R-value is obtained in Table 8, which is also shown in Table 7 represents the simple correlation value, as well as the R square value, which is the coefficient of determination. That is, with the variable that we have in the study we can only predict 17.9% according to the model summary.

Table 9 . ANOVA

ANOVA [a]						
Model		Sum of squares	gl	mean square	F	Next.
1	Regression	26,392	1	26,392	5,452	.028b -
	Residue	121,015	25	4,841		
	Total	147,407	26			

to. Dependent variable: functions

b. Predictors: (constant), priors

Source: self-made

A significance level of 0.28 is shown in Table 9, indicating good significance compared to Table 7. Linear regression analysis (Table 10) reveals a tolerance of 1, which is desirable as it suggests that the *prior knowledge variable* has a unique and non-redundant contribution to the regression model. Therefore, it is highlighted that collinearity is considered present when R squared is greater than 0.9 and the tolerance is below 0.1 (table 8).

Table 10. Coefficients

Coefficients [a]				
Model		95.0% confidence interval for B	Collinearity statistics	
		Upper limit	Tolerance	VIF
1	(Constant)	4,507		
	Previous	1,203	1,000	1,000

to. Dependent variable: Functions

Source: self-made

Discussion

The results of our study indicate that prior knowledge of algebra and arithmetic has a significant impact on the grading of functions in differential calculus. In particular, we observed a positive correlation between prior knowledge of algebra and arithmetic and function grading, suggesting that those students with a stronger understanding of these concepts tend to obtain better grades on function grading. However, it is relevant to note that this correlation does not reach statistical significance, and further data collection is required to validate these results. That is, estimating the independent effects of each variable on the dependent variable is challenging in this context.

Furthermore, our study also revealed a significant relationship between the age of the students and their prior knowledge of algebra and arithmetic. Specifically, older students exhibited tendencies to possess more robust knowledge in these areas compared to their younger counterparts. This phenomenon can be attributed, at least in part, to the greater time and opportunities that older students have had to develop and consolidate their mathematical skills.

It is crucial to highlight that, although there is a relationship between age and prior knowledge, a perfect correlation is not observed. We found that some younger students also had strong prior knowledge of algebra and arithmetic, while some older students had limited knowledge in these areas.

Regarding the sex of the students, we did not identify a significant relationship with the performance grades. Both genders obtained a varied range of scores, indicating that sex is not a determining factor in the ability to learn and master functions in differential calculus.

Likewise, although we did not find a significant relationship between gender and function grades, we also did not find a significant connection between students' gender and their prior algebra and arithmetic knowledge grades. However, it is necessary to recognize that previous studies have reported gender differences in academic performance in various areas of mathematics. These disparities could be due to cultural, social, or educational factors, which may vary in different contexts and student populations. Consequently, it is essential to continue researching and addressing possible gender biases or inequalities in mathematics education.

Together, these results highlight the importance of establishing a solid foundation in algebra and arithmetic to facilitate effective learning of functions in differential calculus.

Contrasting with a more general academic performance statistic, Martínez *et al* . (2014) developed a similar study with a sample of 78 students at the University of Zulia, where they used the expert judgment technique and Cronbach's alpha coefficient. This work showed deficiencies in the prior knowledge of Calculus I students, as well as deficiencies in strategies to reinforce this knowledge, especially in the unit related to functions.

In the case of work like this, mathematics educators and teachers can use these findings to underscore the importance of building a solid foundation in fundamental concepts before tackling more advanced topics in mathematics. Furthermore, these results may be useful in the design of educational interventions and tutoring programs aimed at improving students' understanding of algebra and arithmetic, which would contribute to improving their performance in differential calculus.

Conclusions

The results of our research suggest that prior knowledge in algebra and arithmetic affects the process of learning functions in differential calculus, since students who have a solid base in these areas of knowledge tend to obtain higher grades in functions, although this association does not reach statistical significance.

Furthermore, we observed that the age of the students is related to their performance in prior knowledge since older students tend to obtain higher grades in this area. This phenomenon can be attributed to the progressive development and consolidation of mathematical experience and skills over time, as well as a greater commitment and dedication towards their studies.

On the other hand, we did not identify a significant relationship between the sex of the students and their scores in prior knowledge and functions. This suggests that students' gender does not play a determining role in their ability to acquire and understand the prior concepts essential for learning functions in differential calculus.

Overall, our findings have significant implications for mathematics teaching in the university setting. Consequently, differential calculus instructors could consider the relevance of reinforcing students' prior knowledge in algebra and arithmetic to improve their performance in functions. Likewise, it should be noted that there is no significant relationship between age and function qualification. In other words, the age of the students does not seem to be a determining factor in predicting their performance in office.

In summary, this study contributes to the current body of knowledge regarding the importance of prior knowledge in learning functions in differential calculus and could be of considerable use to teachers and educators dedicated to mathematics instruction at the university level. However, continued research in this area is encouraged to obtain a more comprehensive and detailed understanding of the factors that influence mathematics learning at the university level.

Limitations

Although this study has generated interesting results and has provided valuable information about the interrelationship between prior knowledge in algebra and arithmetic, age, sex, and function grading in the context of differential calculus, it is imperative to recognize the following limitations:

1. Sample size: The sample size is relatively small and may not be representative of the general population. A larger sample could have provided more accurate and reliable results.
2. Lack of diversity: The sample consisted primarily of young college students from a single educational institution, which limits the generalizability of the results to other populations.
3. Selection bias: The students who participated in the study were those who were accessed because they were taught, which may have led to a selection bias.
4. Omitted variables: Although several variables have been considered—such as age, gender, and prior knowledge—other relevant variables—such as socioeconomic status, type of high school students attended, and prior mathematics experience—have not been considered. Were considered. That is, the study was limited only to those studying differential calculus.

Future lines of research

Additional variables that possibly influence the function learning process could be explored in the realm of differential calculus. Among these variables are the motivation and intrinsic interest of the student in the subject, the pedagogical approach used during teaching, as well as the quality of the study materials and teaching resources used. Furthermore, it would be enriching to expand the sample of participants, as well as consider the inclusion of other educational and cultural environments to determine the generalization of the results obtained in this study to various contexts. Longitudinal studies are also suggested to examine the progression of function learning over time and its relationship with students' prior knowledge and age.

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