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Uso de GeoGebra y registros de representación en problemas contextualizados para el aprendizaje de sistemas de ecuaciones lineales 2x2

Use of GeoGebra and representation registers in contextualized problems for learning systems of 2x2 linear equations

Uso do GeoGebra e registros de representação em problemas contextualizados para aprendizagem de sistemas de equações lineares 2x2

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Resumen

Los conceptos matemáticos no son objetos reales y se debe recurrir a distintas representaciones para propiciar el aprendizaje. El uso de GeoGebra para desarrollar conocimiento matemático es reconocido en numerosas investigaciones, las cuales plantean la necesidad del diseño de situaciones que permitan la exploración, formulación de conjeturas, argumentación y evaluación de las mismas. La investigación fue de tipo cualitativa pues interesaba conocer el desarrollo de conocimiento de 25 estudiantes, de un grupo de segundo semestre del turno vespertino de un Bachillerato General por Competencias, alrededor de la resolución de problemas de enunciado verbal y, su representación algebraica y gráfica, que dieran lugar a sistemas de ecuaciones lineales

2x2. La literatura investigada se compone por la teoría de los Registros de representación semiótica de Duval. El análisis de los resultados muestra que fue importante el contexto utilizado, el uso de distintas representaciones y el carácter dinámico del software, para que los estudiantes dieran significado a conceptos como ecuación lineal, incógnita, variable, función, SEL y, por lo tanto, resolvieran los sistemas de ecuaciones lineales 2x2 que subyacían en los problemas.

Palabras clave: Ecuaciones, registros de representación, secuencia didáctica, software, trabajo colaborativo.

Abstract

Mathematical concepts are not real objects and different representations must be used to promote learning. The use of GeoGebra to develop mathematical knowledge is recognized in numerous investigations, which raise the need to design situations that allow exploration, formulation of conjectures, argumentation and evaluation of them. The research was qualitative because it was interested in knowing the development of knowledge of 25 students, from a group of the second semester of the evening shift of a General Baccalaureate by Competencies, around the resolution of verbal statement problems, and their algebraic and graphic representation. that gave rise to systems of 2x2 linear equations. The research literature is composed by Duval's theory of Semiotic Representation Registers. The analysis of the results shows that the context used, the use of different representations and the dynamic nature of the software were important for students to give meaning to concepts such as linear equation, unknown, variable, function, SEL and, therefore, solve the systems of 2x2 linear equations that underlay the problems.

Key words: Equations, representation registers, didactic sequence, software, collaborative work.

Resumo

Os conceitos matemáticos não são objetos reais e diferentes representações devem ser utilizadas para promover a aprendizagem. A utilização do GeoGebra para desenvolver o conhecimento matemático é reconhecida em inúmeras investigações, que levantam a necessidade de desenhar situações que permitam a exploração, formulação de conjecturas, argumentação e avaliação das mesmas. A pesquisa foi qualitativa porque se interessou em conhecer o desenvolvimento do conhecimento de 25 alunos, de uma turma do segundo semestre do turno noturno de um Bacharelado Geral por Competências, em torno da resolução de problemas de enunciados verbais e sua representação algébrica e gráfica. que deu origem a sistemas de equações lineares 2x2. A literatura de pesquisa investigada é composta pela teoria dos Registros de Representação Semiótica de Duval. A análise dos resultados mostra que o contexto utilizado, o uso de diferentes representações e a natureza dinâmica do software foram importantes para que os alunos dessem sentido a conceitos como equação linear, incógnita, variável, função, SEL e, portanto, resolvessem o problema. sistemas de equações lineares 2x2 que fundamentam os problemas.

Palavras-chave: Equações, registros de representação, sequência didática, software, trabalho colaborativo.

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Introduction

Many students face difficulties in understanding the concept of an equation (Arroyo, 2014; Torres and Rodríguez, 2022) and in establishing relationships with other ideas such as variation, unknown and function. Without a solid understanding of these fundamentals, students may experience difficulties in handling and applying algebraic notation, as well as relating it to other representations, such as geometric or tabular. Previous studies (Vargas and Guzmán, 2012; García, 2022) have shown that solving verbally expressed problems, which involve the development of systems of linear equations (SEL), can be a considerable challenge for middle and high school students. The transition from arithmetic to algebra remains an ongoing area of research, as does problem solving that requires the understanding of concepts such as variables, unknowns, functions and equations, supported by technology (Kieran, 2006; Torres and Rodríguez, 2022).

On the other hand, the research literature (Jiménez García and Jiménez Izquierdo, 2017; Sánchez and Borja, 2022) recognizes that the dynamic geometry software GeoGebra can be used for the teaching and learning of algebra. Through this software, students can employ various representations in problem solving and construct meaning of mathematical concepts by transitioning between these representations. Additionally, GeoGebra makes it easy to create a learning environment that encourages activities such as making conjectures, solving problems, and evaluating solutions.

From what was previously mentioned, interest arose in developing a didactic sequence focused on the use of various representations and their conversion, with the purpose of promoting the learning of concepts described above and the resolution of systems of 2x2 linear equations among second-year students. High School semester. Activities were designed that presented contextualized problems, expressed verbally and supported by *applets* developed with GeoGebra, with the objective of facilitating the construction and resolution of systems of 2x2 linear equations, both consistent and inconsistent. This sequence was implemented and the results obtained are presented in this article.

The research questions that guide this article are: What difficulties do students exhibit regarding their knowledge about systems of linear equations? How did the sequence of activities contribute to improving students' knowledge of systems of linear equations and help them solve problems in the context of corn mixtures? The difficulties of students in the second semester of High School were analyzed and how the students performed on problems that involved the conversion between algebraic and graphic representations was monitored.

Research Literature

Duval's Register Theory of Semiotic Representation

Learning a mathematical concept becomes challenging when multiple representations of it are not available. Since mathematical concepts are not tangible, it is essential to use various representations to facilitate their understanding. External representations, known as semiotic representations, encompass constructions of expression and representation systems that can comprise different writing systems (Duval, 1991, as cited in Tamayo, 2006). These semiotic representations constitute the only means of access to mathematical objects, which poses the cognitive challenge of moving from one representation of an object to another representation of the same. Mathematical strategies involve the transformation of semiotic representations (Duval and Sáenz, 2016).

When students work with abstract mathematical concepts, it is essential to use visual and symbolic representations to understand and appropriate the characteristics of these mental objects. According to Duval and Sáenz (2016), in the epistemological context of mathematics, semiotic representations play a central role.

Mathematics learning is a field of study that involves the exploration of essential cognitive activities such as conceptualization, reasoning, problem solving, and text comprehension. These activities demand the use of various registers of representation and expression, in addition to verbal language and images (Duval, 2004, as cited in Oviedo and Kanashiro, 2012).

Semiotic representations are the only means by which mathematical objects can be accessed. According to Duval (1999), the ability to convert at least two representational registers is a crucial indicator in determining whether students have learned a concept. On the other hand, Martínez and Sáez (2014) highlight the importance of using different representations, particularly verbal, algebraic and graphic ones, in the process of learning the topic.

A semiotic system can be a register of representation if it allows three cognitive activities related to semiosis:

1. The presence of an identifiable representation.
2. The treatment of a representation that is the transformation of the representation within the same register where it was formulated.
3. The conversion of a representation that is the transformation of the representation into another representation of another record in which all or part of the meaning of the initial representation is preserved.

GeoGebra

GeoGebra is an easy-to-use, free and open source software that has various features. Available in Spanish, it offers discussion forums and a wiki to share applications through web pages. Using the Java platform, GeoGebra allows students to delve deeper into the fundamentals of school mathematics. It facilitates the quick and easy integration, understanding and application of content from different areas to justify procedures and results (Hernández, 2013).

> The integration of GeoGebra into Mathematics classes has favored the development of students' abilities for experimentation, visualization and recognition of mathematical invariants, as a consequence of the interaction of students with the objects represented in their graphic view . (Prieto, 2016, p. 11)

According to Hohenwarter and Fuchs (2004, as cited in Prieto, 2016), GeoGebra is categorized as a tool for: visualization, discovery construction, and for the representation and communication of knowledge.

GeoGebra facilitates abstraction processes to show how a relationship is built between a geometric model and an algebraic model of a real-life situation, which allows finding mathematical and visual solutions that represent the solution to a given problem (Avecilla et al., 2015).

Dikovic (2009, as cited in Avecilla et al., 2015) showed that GeoGebra offers students the opportunity to develop their intuition through the visualization of mathematical processes. This allows them to explore a wide range of functions by making connections between symbolic and visual representations.

Methodology

The research adopted a qualitative approach, the purpose of which is to understand the phenomena from the perspective of the participants in their natural environment. This approach seeks to examine how individuals perceive and experience the phenomena around them, in order to delve deeper into their points of view, interpretations and meanings. In qualitative research, data collection and analysis are carried out simultaneously.

A didactic proposal is a set of activities designed to achieve specific educational objectives, taking into account a variety of available resources. In this context, the objective is for students to interpret and understand problems expressed in verbal language, formulating the corresponding linear equations and solving these problems with the help of GeoGebra.

Sampling

The research was carried out with a group of 25 students, approximately 16 years old, from a group in the second semester of the evening shift of a General Baccalaureate by Competencies . The students were studying the *Mathematics and Everyday Life II learning unit*, which addresses the topic of SEL.

This learning unit is located in the curricular axis of Mathematical Thinking of the General Baccalaureate by Competencies; for the Common Curricular Framework, with the disciplinary field of Mathematics. The aim is for the student to understand mathematics as part of their daily life, through the use of: estimates, conversions of different systems of units, algebraic language and equations, application of theorems and formulas for the calculation of perimeters, areas and volumes of various forms, as well as the organization and analysis of information from situations in their context; contributing, with this, to the achievement of the graduation profile.

The activities

The sequence was made up of two diagnostic instruments, four activities designed using worksheets and accompanied by GeoGebra *applets* (Figure 1) and an evaluation instrument. Recommendations from Duval's theory of Representation Registers were used for the design of all activities, particularly those that point out the need to encourage the construction of representations and the conversion between these registers. On the other hand, it was taken into account that the activities allowed exploration, formulation of conjectures, argumentation and evaluation thereof; That is, we sought to take advantage of the dynamic nature of the software.

Figure 1. Applet example.

Source: Own elaboration

Diagnostic Activities

Two diagnostic tests were developed which were implemented during the first two class sessions. The objective of the first Diagnostic was to identify the students' prior knowledge and difficulties about concepts related to systems of linear equations with two unknowns (Figure 2). To achieve the stated objective, the diagnosis consisted of 14 questions that addressed different concepts: equation, linear equation, unknown, variable, solution of a linear equation, system of 2x2 linear equations, solution of a system of 2x2 linear equations, solution methods , types of solution of a system of 2x2 linear equations and graphical representation of the solution of a system of 2x2 linear equations.

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Figure 2. Diagnosis 1

1. ¿Qué es una ecuación?
2. ¿Qué es una ecuación lineal?
3. ¿Qué es una incógnita en una ecuación lineal?
4. ¿Qué es una variable?
5. ¿Qué es una solución de una ecuación lineal?
6. ¿Qué es un sistema de ecuaciones lineales 2x2?
7. ¿Qué es una solución de un sistema de ecuaciones lineales 2x2?
8. Menciona algunos métodos de solución de sistemas de ecuaciones lineales.
9. ¿Qué significa que un sistema de ecuaciones lineales 2x2 no tenga solución?
10. ¿Qué significa que un sistema de ecuaciones lineales 2x2 tenga solución única?
11. ¿Qué significa que un sistema de ecuaciones lineales 2x2 tenga infinidad de soluciones?
12. Escribe, para cada inciso, un sistema de ecuaciones lineales 2x2 que cumpla con las siguientes condiciones, y resuélvelo.

a) Un sistema de dos ecuaciones lineales con dos variables, cuya solución sea única.
b) Un sistema de dos ecuaciones lineales con dos variables, cuya solución sea infinita.
c) Un sistema de dos ecuaciones lineales con dos variables en el cual no haya solución.

13. Realiza una representación gráfica que sirva de ejemplo para ilustrar:

a) Un sistema de dos ecuaciones lineales en R^2, con solución única.
b) Un sistema de dos ecuaciones lineales en R^2, con infinidad de soluciones.
c) Un sistema de dos ecuaciones lineales en R^2, sin solución.

14. Escribe al menos 2 ejemplos que se resuelvan con un sistema de ecuaciones lineales:

Source: Own elaboration

The second Diagnosis was made up of 13 questions divided into four sections. Its objective was to identify students' knowledge and skills to interpret linear equations, verbal statements and graphs associated with a problem-situation close to real life, section 1 (Figure 3), where the linear equations underlie. Problems stated in verbal form were established for which students were asked to interpret and write linear equations in algebraic form and vice versa; Situations described graphically were also presented in which students were asked to interpret them.

Figure 3. Diagnosis 2

La tabla proporciona la información de los nutrientes de las distintas variedades de maíz.

	Cantidad de nutrimentos por cada 1000 gramos de producto														
	Calorías	Agua	proteína	grasa	Carbohidratos	Fibra	Ceniza	calcio	Fósforo	hierro	retinol	vit b1	vit b2	vit b5	Ac. Ascórbico
Producto	cal	g	g	g	G	g	g	mg	mg	mg	mcg	mcg	Mcg	mcg	mgg
Maíz amarillo	315	17.2	8.4	1.1	69.4	3.8	1.2	6	267	1.7	2	0.3	0.16	3.25	0.7
Maíz blanco	353	14.1	5.6	4.6	74.3	1.9	1.4	5	249	3	0	0.2	0.16	3	2.6
Maíz choclo	129	67.3	3.3	0.8	27.8	1.5	0.8	8	113	0.8	0	0.14	0.07	1.44	4.8
Maíz morado	357	11.4	7.3	3.4	76.2	1.8	1.7	12	328	0.2	8	0.38	0.22	2.84	2.1

Para hacer una mezcla se utilizan x cantidad en kgs de maíz amarillo, y cantidad en kgs de maíz blanco, z cantidad en kgs de maíz choclo y w cantidad en kgs de maíz morado.

1. De acuerdo con la información de la tabla, escribe la ecuación, expresión algebraica o verbal que falta.

Enunciado verbal	Expresión algebraica
Ecuación que describe la cantidad de kilogramos de maíz choclo y maíz amarillo necesario para obtener una mezcla de 14 gramos de proteínas	
	$1.7x + 0.2w$
Expresión algebraica que describe o permite calcular la cantidad total de calorías que contiene una mezcla compuesta por dos ingredientes: maíz amarillo y maíz morado.	

Source: Own elaboration

Activity 1 "Use of the basic GeoGebra tools"

The objective of Activity 1 was for students to learn how to use GeoGebra through questions that are related to the interpretation of the graphical representation of linear functions with a variable. Some questions related to the corn problem that are presented at the beginning of the activity are shown. activity (Figure 4). This activity was segmented into eight questions; Through the use of GeoGebra *applets* (Figure 5), students explored the assignment of different values to each variable and analyzed the behavior of the resulting values in algebraic expressions and linear equations according to the data described in a board.

Figure 4. Activity 1

1. Selecciona la opción "Punto en objeto". Da clic en algún punto de la recta $f(x) = 14.1x$. Observa que aparece un par ordenado (x , y) del lado izquierdo (en la vista algebraica). ¿Qué significan estos pares ordenados en términos del problema del maíz?

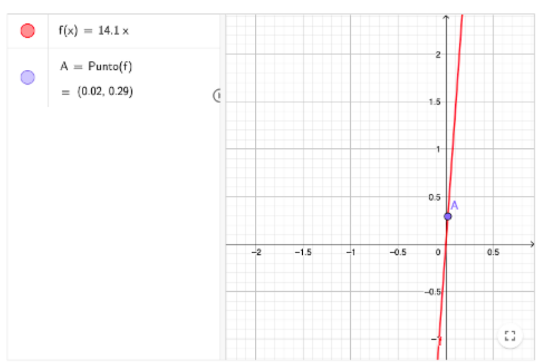
2. Con el puntero del ratón, arrastra el punto A en la recta. ¿Cambian de valor los pares ordenados? ¿Por qué? ¿Cómo se pueden interpretar estos valores?

3. ¿Cuántos gramos de agua corresponden a 1.5 kg de maíz blanco? ¿Cuántos gramos de agua corresponden a 3 kg de maíz blanco? ¿Cuántos gramos de agua corresponden a 5 kg de maíz blanco?

4. ¿Cuántos kilogramos de maíz blanco corresponden a 3 gramos de agua? ¿Cuántos kilogramos de maíz blanco corresponden a 6 gramos de agua? ¿Cuántos kilogramos de maíz blanco corresponden a 10 gramos de agua?

Source: Own elaboration

Figure 5. GeoGebra Applet



Source: Own elaboration

Activity 2 "Linear equations with one and two unknowns"

The concepts explored by students in Activity 2 were algebraic expression, linear function and linear equation with two unknowns (Figure 6). It was designed for students to make conversions between verbal, graphical, or algebraic representations. In this activity, students had to interpret the results, expressing them in terms of the corn problem instead of referring only to the mathematical symbols that represent the variables.

Figure 6. Activity 2

La tabla proporciona la información de los nutrientes de las distintas variedades de maíz.

	Cantidad de nutrimentos por cada 1000 gramos de producto														
	Calorías	Agua	proteína	grasa	Carbohidratos	Fibra	Ceniza	calcio	Fósforo	hierro	retinol	vit b1	vit b2	vit b5	Ac. Ascórbico
Producto	cal	g	g	g	G	g	g	mg	mg	mg	mcg	mcg	Mcg	mcg	mgg
Maíz amarillo	315	17.2	8.4	1.1	69.4	3.8	1.2	6	267	1.7	2	0.3	0.16	3.25	0.7
Maíz blanco	353	14.1	5.6	4.6	74.3	1.9	1.4	5	249	3	0	0.2	0.16	3	2.6
Maíz choclo	129	67.3	3.3	0.8	27.8	1.5	0.8	8	113	0.8	0	0.14	0.07	1.44	4.8
Maíz morado	357	11.4	7.3	3.4	76.2	1.8	1.7	12	328	0.2	8	0.38	0.22	2.84	2.1

Para hacer una mezcla se utilizan x cantidad en kgs de maíz amarillo, y cantidad en kgs de maíz blanco, z cantidad en kgs de maíz choclo y w cantidad en kgs de maíz morado.

Describe lo que significan las siguientes expresiones y ecuaciones en el contexto del problema del maíz

1. ¿Qué significan $69.4x, 14.1y, 67.3z, 11.4w$?
2. Grafica en GeoGebra la función $f(x) = 69.4x$, asigna valores distintos a x y calcula los valores que se obtienen para cada valor de x. ¿Qué significan los valores calculados? ¿Qué tipo de gráfica se produce con los valores asignados y los obtenidos por los cálculos?
3. ¿Qué significan las ecuaciones $69.4x = 20, 69.4x = 32, 69.4x = 42.3$ en el contexto del problema del maíz? ¿Cuál es el valor de la incógnita x para cada ecuación?

Source: Own elaboration

Activity 3. "Systems of linear equations in GeoGebra"

In Activity 3, the objective was for students to analyze, interpret and solve with GeoGebra, a system of 2x2 linear equations algebraically and graphically from a verbal statement (Figure 7). The concept studied in this activity was SEL with two unknowns. The students had to convert the verbal representation to the algebraic representation and then to the graph with the help of GeoGebra, to finally interpret the results obtained in terms of the corn problem.

In each of the problems posed, the teacher provided a detailed description of the situation to be solved. In three of these problems, students were asked to write a 2x2 system of linear equations (SEL), with the particularity that in one of them the answer made no sense in the context of the corn problem (Figure 7), given that the solutions corresponding to the quantities of the ingredients in the mixture were negative. In another problem, a situation arose that required calculating a 2x2 SEL with infinite solutions, while in a third problem, the challenge was to solve a 2x2 SEL that had no solution. This activity was designed with the objective of providing students with the opportunity to deepen their understanding of the concepts related to systems of linear equations with two unknowns, addressing cases with a single solution, without a solution, and with infinite solutions.

Figure 7. Activity 3. Two of five sections of the problem used.

La tabla proporciona la información de los nutrientes de las distintas variedades de maíz.

	Cantidad de nutrimentos por cada 1000 gramos de producto														
	Calorías	**Agua**	**proteína**	**grasa**	**Carbohidratos**	**Fibra**	**Ceniza**	**calcio**	**Fósforo**	**hierro**	**retinol**	**vit b1**	**vit b2**	**vit b5**	**Ac. Ascórbico**
Producto	**cal**	**g**	**g**	**g**	**G**	**g**	**g**	**mg**	**mg**	**mg**	**mcg**	**mcg**	**Mcg**	**mcg**	**mgg**
Maíz amarillo	315	17.2	8.4	1.1	69.4	3.8	1.2	5.5	267	1.7	2	0.3	0.16	3.25	0.7
Maíz blanco	353	14.1	5.6	4.6	74.3	1.9	1.4	5.5	249	3	0	0.2	0.16	3	2.6
Maíz choclo	129	67.3	3.3	0.8	27.8	1.5	0.8	8	113	0.8	0	0.14	0.07	1.44	4.8
Maíz morado	357	11.4	7.3	3.4	76.2	1.8	1.7	12	328	0.2	8	0.38	0.22	2.84	2.1

Para hacer una mezcla se utilizan *x* cantidad en kgs de maíz amarillo, *y* cantidad en kgs de maíz blanco, *z* cantidad en kgs de maíz choclo y *w* cantidad en kgs de maíz morado.

1. Con la información de la tabla escribe el sistema de ecuaciones lineales 2x2 que representa la solución de cada uno de los problemas siguientes. Con el apoyo de GeoGebra resuelve los sistemas de ecuaciones lineales, elabora la gráfica y realiza la interpretación gráfica. Interpreta los resultados.
   a. ¿Qué cantidad de kgs de maíz amarillo y kgs de maíz blanco se requiere si se desea elaborar una mezcla con esos ingredientes de manera que la mezcla tenga: 2430.9 calorías y 20.78 gramos de grasa?
   b. ¿Qué cantidad de kgs de maíz choclo y kgs de maíz morado se requiere si se desea elaborar una mezcla con esos ingredientes de manera que la mezcla tenga: 35.2 gramos de proteína y 188 gramos de agua?

Source: Own elaboration

Activity 4 "Solution of word problems that involve the construction of systems of linear equations in GeoGebra"

Activity 4 consisted of a series of word problems that involved the construction of an SEL with two variables and its solution through GeoGebra. The objective was for students to analyze, interpret and solve word problems in a numerical, graphic or algebraic way. The main characteristic of these problems is that they were not within the context of corn mixtures (Figure 8). Students had to transfer, therefore, their knowledge to the resolution of problems stated in other contexts.

In this activity, students had to solve SEL 2x2 with a single solution, without a solution or with an infinite number of solutions; In this way they would be able to move from the verbal representation to the algebraic representation and, subsequently, to the graphic representation for interpretation.

Figure 8. Activity 4. Three of seven problems used.

1. Los siguientes problemas representan actividades de la vida cotidiana, contesta en cada uno lo que se te pide.

a. Una persona compra una mezcla de café de 3.25 kg por un precio de $225. Si el kilogramo de café Córdoba cuesta $80 y el Xalapa $60, ¿cuánto lleva de cada tipo de café la mezcla?
b. El señor López se ejercita diariamente, cierto día corre 30 minutos y nada 30 minutos recorriendo una distancia total de 8 km, al día siguiente corre 45 minutos y nada 15 minutos para un total de 10 km, si su velocidad en cada deporte es la misma en ambos días, ¿cuál es la velocidad con la que corre y cuál con la que nada?
c. Un granjero prepara una mezcla de avena y maíz para alimentar a su ganado. Cada kilogramo de avena contiene 0.15 kg de proteína y 0.6 kg de carbohidratos, mientras que cada kilogramo de maíz contiene 0.1 kg de proteína y 0.75 kg de carbohidratos. ¿Cuántos kilogramos de cada uno pueden usarse para cumplir con los requerimientos nutricionales de 7.5 kg de proteínas y 50 kg de carbohidratos por comida?

Source: Own elaboration

Evaluation Instrument

In the Evaluation, the objective was to know what the students had learned regarding algebraic expressions, functions, linear equations, and solving systems of 2x2 linear equations with the support of GeoGebra. This instrument served as a final evaluation of the implementation of the proposal; some of the questions included are shown (Figure 9).

A total of 24 students participated in the Evaluation, since one of the students could not report to school due to medical complications. They showed their knowledge regarding algebraic expressions, functions, linear equations, SEL 2x2 with and without the support of GeoGebra for the solution of SEL 2x2; This instrument was used as a final evaluation of the implementation of the sequence of activities.

This evaluation consisted of nine questions, the first five were multiple choice and the last four were open-ended in which students had to describe the type of system solution proposed.

Figure 9. Evaluation.

1. De las siguientes afirmaciones con respecto a la solución de un sistema de ecuaciones lineales 2x2, ¿cuál de ellas no es verdadera?
   a) Es un par ordenado que satisface ambas ecuaciones.
   b) Su gráfica consiste en el punto de intersección de las gráficas de las ecuaciones.
   c) Los sistemas de ecuaciones lineales de 2x2 pueden tener dos soluciones, una se representa por x y otra por y.
   d) Si el sistema es inconsistente, no existe una solución.

2. ¿Cuál de las siguientes afirmaciones es cierta para un sistema de ecuaciones lineales 2x2 en el que sus rectas jamás se intersectan?
   a) No existe una solución.
   b) La gráfica del sistema está sobre el eje y.
   c) La gráfica de la solución es una recta.
   d) La gráfica de la solución es el punto de intersección de dos líneas.

3. ¿Cuál de las aseveraciones que siguen es cierta para el siguiente sistema de ecuaciones?

$$3x - 2y = 6$$

$$6x + y = 12$$

   a) El sistema no tiene solución.
   b) La solución es (-1,2)
   c) La solución se encuentra sobre la recta $x=2$
   d) Las ecuaciones son equivalentes.

Source: Own elaboration

Data collection

The two diagnostic exams and the final evaluation were carried out individually by the students, the other activities of the didactic proposal were solved in bins. The reason they were done in bins was so that students could discuss their GeoGebra explorations and conjectures, as well as have the opportunity to argue and evaluate their results. This in turn would allow more information about their solution processes.

In the activities that were carried out in GeoGebra, the researcher-teacher, author of this article, asked the students to record all their actions carried out on the computer with the Windows PSR command, in addition to this he took note of what they I observed every activity; He also asked the students to audio record the entire conversation between them while they solved each of the activities, which were submitted. After the students solved each of the activities, a feedback session was held.

Data processing

The triangulation method was used (Hernández, Fernández and Baptista, 2014), which consists of crossing information from various sources of information, these were the sheets of the didactic sequence, interviews, video and field diary.

The responses written on the worksheets were analyzed to observe the students' actions and reflections when using different registers and performing conversions. The videos of the sessions were analyzed to capture details not captured with the other instruments.

Results

This section contains the results of the analysis of the learning achieved in each of the activities carried out. The learning achieved by the group is also exemplified through the results of an average student (E13) and the bin to which he belonged (EQ8).

Diagnosis 1

The results obtained allowed us to identify that the majority of students conceptualize an equation as an operation used to find missing data. Some students defined an equation as an equality between two expressions.

In relation to the concept of a linear equation, the students interpreted it as an equation written horizontally or as an equation without negative exponents. Additionally, they deduced that linear equations were those that were not quadratic, possibly due to their limited experience with these types of equations in previous high school courses.

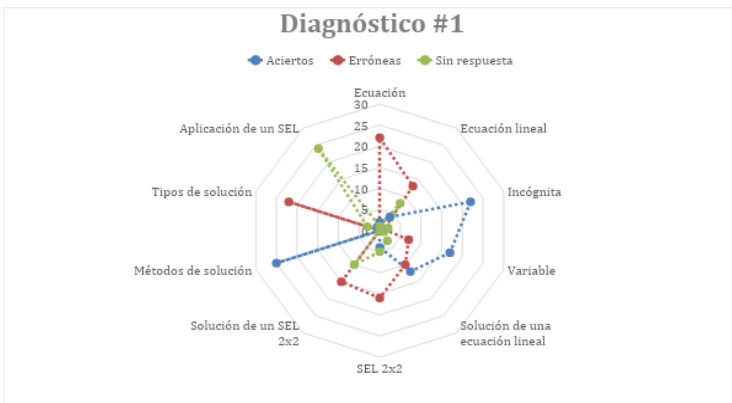
In the diagnosis, it was observed that more than 90% of the students did not respond adequately to the concepts related to the graphical representation and application of systems of linear equations, including the algebraic representation of a system of 2x2 linear equations with infinite solutions. These findings are summarized in Figure 10.

All students recognized the existence of different solution methods, although the Addition and Subtraction method was the most mentioned. However, they showed lack of knowledge about the application and usefulness of systems of linear equations in everyday situations.

Regarding the concept of a linear equation, 84% of the students responded with terms such as simple equation or equation without fractions. Furthermore, approximately 40% of the students were unable to adequately define concepts such as unknown, variable, and solution of an equation.

The graphical representation of a system of 2x2 linear equations was not understood by the students, which agrees with the findings of Duval (1996), who pointed out that students often have difficulty interpreting a system of linear equations graphically.

Figure 10. Diagnostic results 1.



Source: Own elaboration

An illustrative example of the procedures followed by student E13 is provided, presented in Figure 11. This student managed to define concepts such as equation, variable, linear equation and solution methods for systems of 2x2 linear equations. However, he did not demonstrate knowledge about the types of solutions of systems of 2x2 linear equations or the forms of graphical representation thereof. He left blank the answers to the questions that required writing and solving a system of 2x2 linear equations.

Figure 11. Diagnosis 1 of student E13.

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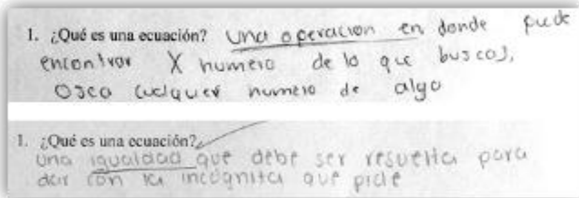

1. ¿Qué es una ecuación? Una operacion en donde puede encontrar X numero de lo que buscas, Osea cualquier numero de algo

1. ¿Qué es una ecuación? Una igualdad que debe ser resuelta para dar con la incognita que pide

Source: Own elaboration

Diagnosis 2

The students were presented with the same situation as in the initial diagnosis. It was observed that questions related to graphical representation were those that generated the most omissions by students, while questions that involved conversions between algebraic and verbal representations were the most answered.

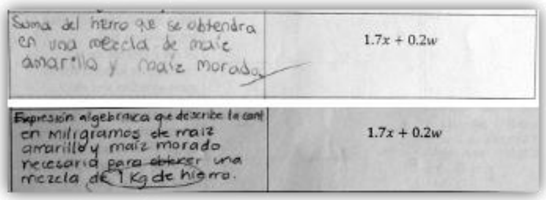
Regarding the conversions between representations (Figure 12), it was found that the students were more successful in the conversion from the verbal to the algebraic representation, while they faced greater difficulties in the conversion from the algebraic to the verbal representation (Figure 13) . Additionally, there were a significant number of questions related to graphical representation that were left unanswered.

Conversions between verbal and algebraic representations were the least problematic for students, while graphical interpretation of a linear function was understood by approximately 50% of students. However, the graphical interpretation of the solution of a system of 2x2 linear equations represented a challenge for the students (Figure 14), since they were only able to adequately interpret the single solution cases. In the other cases, they found it difficult or did not know how to interpret the results. Despite these difficulties, students demonstrated skills in analyzing data presented in table form and manipulating it, suggesting a good level of competence in this aspect.

Figure 12. Diagnostic results 2.

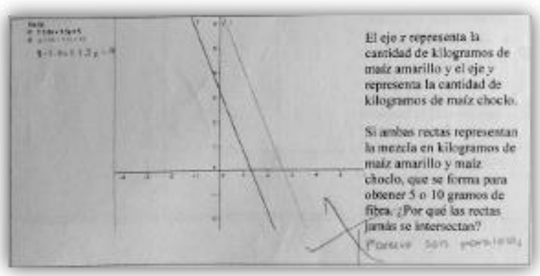
Source: Own elaboration

Figure 13. Conversion from algebraic to verbal representation.



Source: Own elaboration

Figure 14. Interpretation of the graphical solution of a 2x2 SEL.

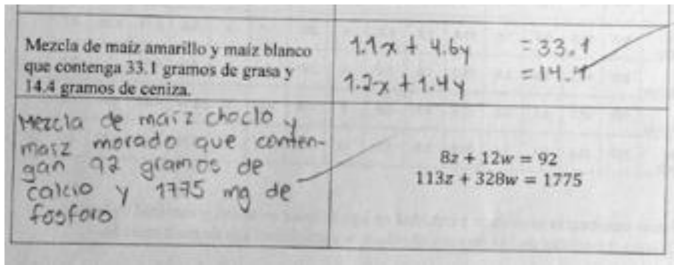


Source: Own elaboration

The procedures of the average student E13 are presented, detailed in Figure 15.

This student demonstrated the ability to make conversions between verbal and algebraic representations, as well as vice versa. Furthermore, he correctly interpreted the points on a line within the context of the corn mixture problem. However, he faced difficulties when interpreting the graphs of a system of linear equations without a solution and with infinite solutions, leaving the corresponding answer blank.

Figure 15. Conversion between verbal and algebraic representations.



Mezcla de maíz amarillo y maíz blanco que contenga 33.1 gramos de grasa y 14.4 gramos de ceniza.	1.1x + 4.6y = 33.1 1.2x + 1.4y = 14.4
Mezcla de maiz choclo y maiz morado que conten-gan 92 gramos de calcio y 1775 mg de fosforo	$8z + 12w = 92$ $113z + 328w = 1775$

Source: Own elaboration

In summary, the results obtained from the application of this instrument indicate that students still face difficulties in interpreting the graphical solution of a system of 2x2 linear equations. However, a significant improvement was observed in students' ability to make conversions between verbal and algebraic representation, and vice versa.

Activity 1. Use of the basic GeoGebra tools

The main concept addressed by the students in this activity was that of a linear function. Conversion between various representations was encouraged when solving exercises involving graphical, algebraic, verbal, and tabular representations. Through the *applet* (Figure 5), students were able to assign meaning to each point on the line of the linear function in the context of the problem. The use of sliders allowed them to dynamically access each point on the line, thus facilitating their understanding. The following results were observed:

- In 47% of the responses, binas converted between representations of a linear function by alternating between verbal, algebraic, and graphical representations.
- The interpretation of the relationships in the corn problem was the least problematic part for the students, with only 8% presenting difficulties.
- Interpreting ordered pairs was the most difficult part for students, with 75% failing to describe each of the ordered pairs according to the given context.
- In questions that explored the concept of a linear function through graphical representation, many students left their answers blank.

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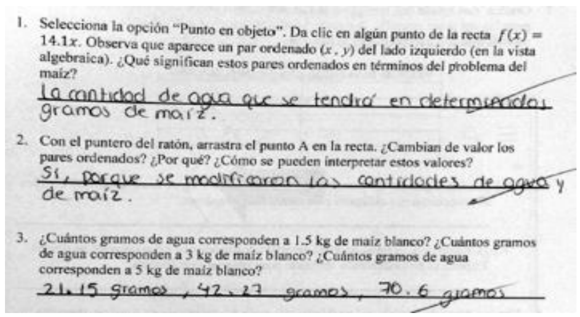
- The main challenge for students regarding the use of GeoGebra was not familiarization with the software itself, but rather the construction of linear functions for implementation in it.

The procedures of the average bin EQ8, composed of students E13 and E20, are shown in figure 16. It could be observed that the bin was able to interpret the linear functions according to the context of the corn problem, but had difficulties in constructing a function from the data presented in the corn nutrient table, leaving that answer blank.

Figure 16. Responses of the EQ8 bin.



1. Selecciona la opción "Punto en objeto". Da clic en algún punto de la recta $f(x) = 14.1x$. Observa que aparece un par ordenado (x, y) del lado izquierdo (en la vista algebraica). ¿Qué significan estos pares ordenados en términos del problema del maíz?

   La cantidad de agua que se tendrá en determinados gramos de maíz.

2. Con el puntero del ratón, arrastra el punto A en la recta. ¿Cambian de valor los pares ordenados? ¿Por qué? ¿Cómo se pueden interpretar estos valores?

   Sí, porque se modificaron las cantidades de agua y de maíz.

3. ¿Cuántos gramos de agua corresponden a 1.5 kg de maíz blanco? ¿Cuántos gramos de agua corresponden a 3 kg de maíz blanco? ¿Cuántos gramos de agua corresponden a 5 kg de maíz blanco?

   21.15 gramos, 42.27 gramos, 70.6 gramos

Source: Own elaboration

Activity 2. Linear equations with one and two unknowns

Students were able to make conversions between the verbal, graphical, and algebraic representations, interpreting the results in the context of the corn problem instead of focusing solely on the mathematical symbols that represented the variables. The data collected revealed that graphic representation continued to be the one that presented the greatest difficulties for students (Figure 17).

The concept of a linear equation with two unknowns turned out to be the most challenging for 58% of the students. One of the obstacles encountered by the teams was the lack of interpretation according to the specific data of the corn problem.

On the other hand, the concept of algebraic expression turned out to be the least difficult, since all the teams managed to interpret the expressions based on the context of the corn problem. The students demonstrated ease in interpreting an algebraic expression from the data presented in a table. However, when faced with the graphical representation of a linear function, 33% of the

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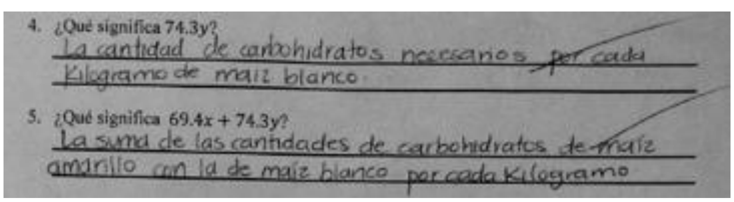
students encountered difficulties when solving the problems, as they were unable to understand what exactly the graphs represented.

Figure 17. Successes and errors classified according to the concepts addressed in the questions

Source: Own elaboration

Figure 18 shows the interpretations made by the EQ8 bin, related to the corn problem, with respect to algebraic expressions with one unknown. Using GeoGebra as a tool, they graphed the functions and assigned values to the independent variable, achieving an accurate interpretation. Linear equations with one unknown and algebraic expressions with two unknowns were understood correctly. Finally, they performed an adequate conversion from the algebraic to the verbal representation.

Figure 18. Interpretation of algebraic expressions of an unknown.



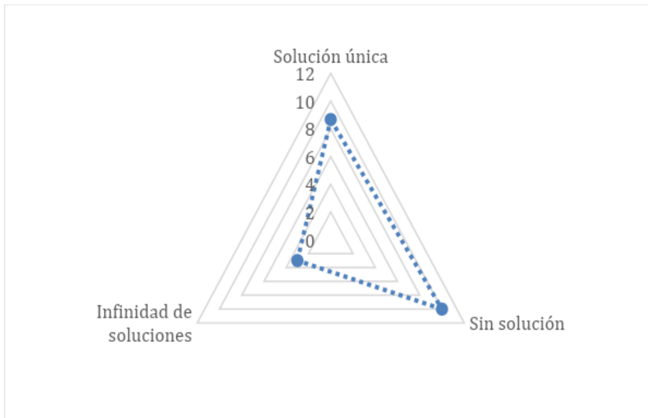
Source: Own elaboration

Activity 3. SEL with GeoGebra

Students faced the analysis, interpretation and resolution of systems of 2x2 linear equations using GeoGebra, both in algebraic and graphical form, starting from verbal statements.

It was observed that the system of 2x2 linear equations that presented the greatest difficulty for 92% of the students was the one with infinite solutions, while the system with the highest percentage of correct answers, 83%, was the SEL 2x2 without solution (Figure 19). This was because the students used the graphical representation of GeoGebra to address the algebraically posed systems; They found it more intuitive to interpret two parallel lines than two overlapping lines. It is relevant to mention that the students were not able to obtain the solutions of the SEL 2x2 with infinite solutions; They simply indicated that there was no solution, noting that it was only a single line and lacked an intersection.

Figure 19. Average of correct answers in the 2x2 SEL.



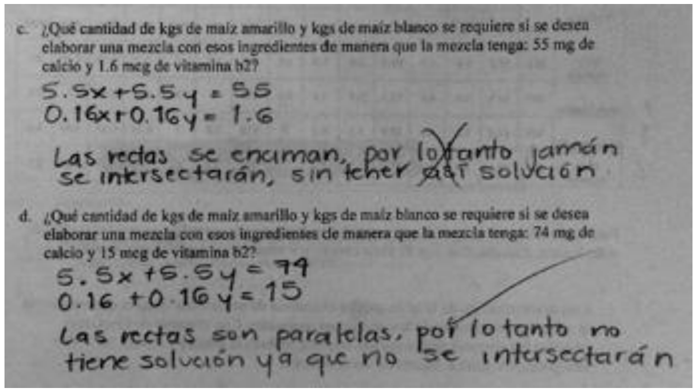
Source: Own elaboration

In this activity, students answered all the questions for the first time, demonstrating an improvement in their abilities to interpret verbally stated problems. This improvement was especially observed when the conversion was carried out from the verbal record to the graphic (geometric) record using GeoGebra, through the algebraic language as a bridge.

The process followed by the students initially involved translating verbal language into algebraic language when constructing the equations. Once the equations were obtained, they used GeoGebra to graph them and find the solution. Finally, they interpreted the results obtained in terms of the situations presented.

The procedures of the average student E13, in his bin EQ8, are detailed in Figure 20. The bin erroneously selected the data from the table in one of the problems and failed to interpret the solution of an SEL when it involved infinite solutions. In all the problems of the activity, the bina made the transition from verbal to algebraic representation. Then, using GeoGebra, he converted the algebraic representation to a graphical representation and finally returned to the verbal representation.

Figure 20. Representation conversion and interpretation of results.


c. ¿Qué cantidad de kgs de maíz amarillo y kgs de maíz blanco se requiere si se desen elaborar una mezcla con esos ingredientes de manera que la mezcla tenga: 55 mg de calcio y 1.6 mcg de vitamina b2?

5.5x+5.5y = 55
0.16x+0.16y = 1.6

Las rectas se enciman, por lo tanto jamán se intersectarán, sin tener así solución

d. ¿Qué cantidad de kgs de maíz amarillo y kgs de maíz blanco se requiere si se desea elaborar una mezcla con esos ingredientes de manera que la mezcla tenga: 74 mg de calcio y 15 mcg de vitamina b2?

5.5x +5.5y = 79
0.16 +0.16y = 15

Las rectas son paralelas, por lo tanto no tiene solución ya que no se intersectarán

Source: Own elaboration

Activity 4. Solution of word problems that involve the construction of SEL 2x2 in GeoGebra

Students analyzed, interpreted and solved word problems using different numerical, graphical and algebraic approaches. Of the six problems posed, three involved systems of linear equations with a unique solution, one presented infinite solutions and the remaining two had no solution. 83% of the students were able to successfully solve the SELs with a single solution, while 58% were able to correctly address the problems without a solution. However, only 41% were able to adequately solve problems involving infinite solutions (Figure 21).

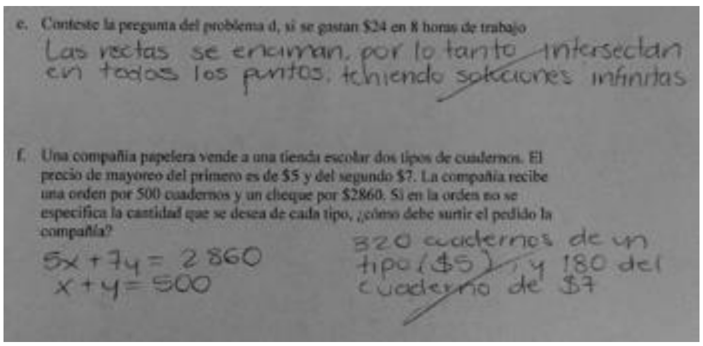
A crucial aspect in all the problems of this activity was the realization of conversions between the different representations. The problems were initially presented in the form of verbal statements, then the students converted to algebraic representations and later to graphical representations. Finally, they interpreted the results obtained in terms of the problem posed.

Figure 21. Results of Activity 4.

Source: Own elaboration

Some procedures of the average student E13, within his EQ8 bin, are shown in figure 22, where his ability to convert between different representations and to interpret the results in accordance with the proposed context is evident. However, in indeterminate systems of consistent linear equations, he simply mentioned the existence of an infinite number of solutions, without attempting to find values that could solve the system.

Figure 22. Conversion between different representations.

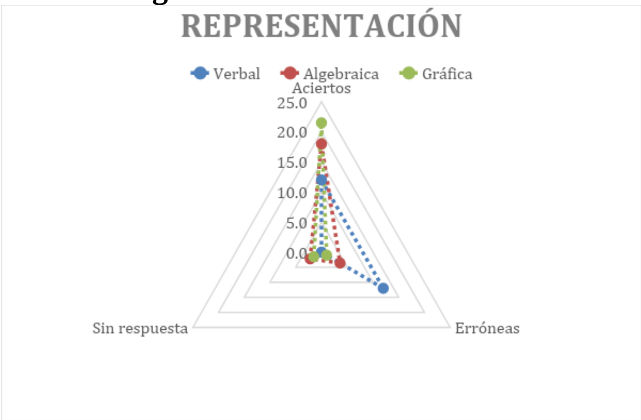


Source: Own elaboration

Assessment

The questions related to the graphical representation of a 2x2 SEL (questions 5, 6a, 6b, Figure 24) showed a high level of correct answers, with 90% correct answers according to the results (Figure 23). This finding is of great importance, since at the beginning of the proposal, during the initial diagnoses, the graphic representation was the aspect that presented the greatest difficulties for the students.

Figure 23. Evaluation Results.



Source: Own elaboration

One of the questions (question 6b, Figure 24), despite being an algebraic representation, was addressed by the students by converting the algebraic representation to graphical representation with the assistance of GeoGebra.

When examining the performance of student E13, it is observed that he solved the 2x2 SELs represented algebraically by converting them to graphical representation. In addition, he interpreted the different types of SEL solutions, both in their algebraic and graphical forms. Although he was not able to define the concept of a 2x2 SEL solution during the diagnosis, he was able to apply it in the evaluation, using graphical representation to solve the problems (Figure 24).

A detailed analysis reveals that student performance improved as the teaching proposal progressed. Furthermore, it is evident that collaborative work influenced the results of Activity 4 and the Evaluation: while they correctly solved Activity 4, they faced certain difficulties in the Evaluation. The individual contribution of each student to teamwork is also highlighted; The best performance was demonstrated by E10, who was part of EQ5, the leading team in overall performance.

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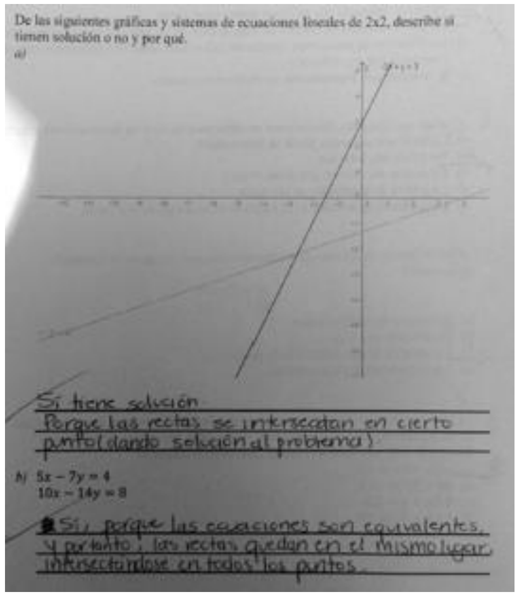
Figure 24. Student evaluation E13



Source: Own elaboration

The results derived from the evaluation instrument indicate a significant improvement in the students' ability to use various forms of representation and to interpret the results obtained when solving the problems posed. In this sense, following the perspective of Duval (2004), who highlights that effective learning of mathematics involves the use of different registers of representation and expression, we can conclude that students have strengthened their understanding of systems of linear equations.

After the application of the evaluation instrument, a series of interviews were carried out with the students. According to the opinions expressed by the students themselves, the dynamics of the teaching sequence implemented was perceived as innovative. This methodology, which combined a family context, the use of technology, the practical application of mathematical concepts and collaborative work in teams, departed from traditional approaches and generated a high degree of interaction and commitment on the part of the students.

Discussions

The guiding research questions of this study are: What difficulties do students exhibit regarding their knowledge about systems of linear equations? How did the sequence of activities contribute to improving students' knowledge of systems of linear equations and help them solve problems in the context of corn mixtures?

Regarding the first question, the analysis of the responses obtained revealed that students initially showed confusion when distinguishing between an algebraic expression and a linear equation, particularly when interpreting the variables during the conversion from the verbal to the algebraic representation. Regarding the second research question, it was observed that the sequence of activities contributed to the improvement of students' knowledge about systems of linear equations since the implementation of the first activity. Throughout the process, the students progressively managed to interpret the variables and linear equations associated with the problem situation of corn mixtures, thanks to the support provided by the GeoGebra applets. These applets allowed them to explore variation between variables, formulate conjectures, and evaluate them in the context of the problem. However, they could not identify solutions to an inconsistent system, which coincides with similar findings reported by Pérez and Vargas (2019).

The sequence based on the corn problem-situation, supported by GeoGebra *applets* , encouraged students to learn concepts associated with systems of 2x2 linear equations, solution and interpretation processes of these systems, and the construction and conversion between representation records. The above, based on Duval (1996), who points out that "to learn a mathematical concept one must go through the treatment and conversion of different registers of semiotic representation." Something that stands out is how the students were also able to interpret the results of the solution of a 2x2 SEL. They were not only limited to solving the proposed 2x2 SEL, but also to interpreting results in terms of a real-life situation. This sequence of activities was essential for understanding the solution processes, as happened in the study by Segura (2004) in which students moved from one representation to another to solve the situations posed.

One of the limitations of this research is that it was not taught how to find solutions when it was a 2x2 SEL with an infinite number of solutions, it was only indicated that it had infinite solutions.

Conclusions

The concept of unknown was the most familiar to 88% of the students, while the addition and subtraction method stood out as the best-known method for solving 2x2 systems of linear equations (SEL), mentioned by 72% of the students. participants. However, the concepts of linear equation and SEL 2x2 presented greater difficulties, since 84% of students had difficulty defining them. Additionally, students faced difficulty graphically representing a 2x2 SEL, which reflected a limitation in their ability to use and interpret graphical representations.

The use of GeoGebra facilitated the transition between the different representations of the 2x2 systems of linear equations, including verbal, algebraic and graphical representations. This tool allowed the resolution of systems of consistent and inconsistent 2x2 linear equations, which were linked to problems posed verbally. Additionally, GeoGebra supported students in solving problems in the context of corn mixtures and other similar contexts.

The pair work approach and the use of GeoGebra *applets* during the activities allowed students to progressively develop their knowledge and skills. This approach promoted the communication of ideas among students, as well as the ability to modify and refine procedures throughout the learning process.

The responses to the activities and the evaluation of the didactic proposal provide solid evidence that the students of the second semester of the High School have significantly improved their understanding and skills in solving 2x2 systems of linear equations (SEL), both consistent and inconsistent. as well as in solving problems posed verbally. This progress is evident in their ability to make fluid conversion between verbal, algebraic, and graphical representations, and in their ability to interpret the results in the specific context of the situation presented. These achievements are supported by Duval and Sáenz's (2016) Theoretical Framework of Registers of Semiotic Representation, which provides a solid framework for understanding students' mathematical learning process.

Future lines of research

Based on the results obtained, several lines of research can be proposed for future studies:

- To investigate effective strategies to address students' difficulties in distinguishing between algebraic expressions and linear equations during conversion from verbal to algebraic representations. This could include designing and implementing specific teaching activities focused on understanding these differences.

- Explore how different pedagogical approaches, including the integration of educational technologies such as GeoGebra, can contribute to the development of students' knowledge of systems of linear equations and their ability to solve problems in specific contexts.
- Address limitations identified in this research, such as the lack of teaching on solving systems of linear equations with infinite solutions. It would be important to design specific interventions to teach students to address these types of situations and understand their meaning in real contexts.

Future research could focus on effective teaching strategies, innovative pedagogical approaches, the role of educational technologies, the application of mathematical concepts in real situations and overcoming the limitations identified in this study.

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Conceptualization	César Eduardo (same) Veronica (same)
Methodology	Veronica (main) César Eduardo (supports)
Software	Cesar Eduardo
Validation	Veronica
Formal Analysis	Cesar Eduardo
Investigation	Cesar Eduardo
Resources	César Eduardo (same) Veronica (same)
Data curation	César Eduardo (same) Veronica (same)
Writing - Preparation of the original draft	Cesar Eduardo
Writing - Review and editing	Veronica
Display	César Eduardo (main) Veronica (supports)
Supervision	Veronica (main) César Eduardo (supports)
Project Management	César Eduardo (same) Veronica (same)
Fund acquisition	Cesar Eduardo