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Scientific articles

El sistema de prácticas del enfoque ontosemiótico en plataforma Moodle para la enseñanza de interpolación polinomial

The ontosemiotic approach practice system in Moodle platform for teaching Polynomial Interpolation

O sistema de práticas da abordagem ontossemiótica na plataforma Moodle para o ensino da interpolação polynomial

Teresa Carrillo Ramírez
Universidad Autónoma de Querétaro, México
teresacr71@comunidad.unam.mx
https://orcid.org/0000-0002-8728-5824

María del Carmen González Videgaray
Universidad Nacional Autónoma de México, México
mcgv@unam.mx
https://orcid.org/0000-0003-4707-3701

Sandra Luz Canchola Magdaleno
Universidad Autónoma de Querétaro, México
sandra.canchola@uaq.mx
https://orcid.org/0000-0002-7497-281X

Resumen

La asignatura Métodos Numéricos es obligatoria en las carreras de matemáticas e ingeniería debido a que es fundamental para muchos desarrollos computacionales actuales. Sin embargo, aunque su aprendizaje requiere que los estudiantes posean un conocimiento matemático adecuado para comprender y aplicar estos métodos, la enseñanza suele centrarse en la mecanización, lo que conduce a aprendizajes insuficientes. En este contexto, el enfoque ontosemiótico (EOS) (Godino *et al*., 2007) es un sistema teórico que integra diversas teorías de didáctica de la matemática, y supone que la actividad numérica se centra en la resolución de problemas. Por tanto, en este estudio se propuso construir y probar una ruta tecnopedagógica basada en el sistema de prácticas del EOS implementada en Moodle con el fin de desarrollar el conocimiento matemático necesario para el aprendizaje y aplicación de métodos de interpolación polinomial. Para ello, se siguió la metodología de investigación-acción con el propósito de desarrollar materiales y actividades, y seleccionar problemas contextualizados disponibles en la plataforma Moodle. Los estadísticos generados por la plataforma ayudaron a validar la ruta propuesta, y se realizó un análisis cuantitativo de las calificaciones y registros en Moodle, junto con un análisis cualitativo de la percepción de los estudiantes. Los resultados muestran que el sistema de prácticas del EOS implementado en Moodle mejora el aprendizaje matemático, con una tasa de aprobación del 86 %. Asimismo, los estudiantes destacaron que los problemas contextualizados fueron el elemento que más contribuyó a su aprendizaje. Por tanto, se puede sugerir que esta propuesta podría generalizarse para la enseñanza de matemáticas computacionales en educación superior.

Palabras clave: semiótica, educación matemática, tecnopedagógico, métodos numéricos, educación superior.

Abstract

The subject of numerical methods is a must in mathematics and engineering careers; its formative practice is the substantial basis of many of the current computational developments. Its learning requires that the student has the mathematical knowledge that allows him/her to understand and apply them, however, its teaching is usually focused on mechanization, producing insufficient learning. The Ontosemiotic Approach (EOS) (Godino et al., 2007) is a theoretical system that integrates various theories of Didactics of Mathematics, which assumes that mathematical activity is focused on problem solving. The objective of this work was to build and test a technopedagogical route based on the EOS practice system implemented in Moodle to achieve the mathematical knowledge required for the learning and application of polynomial interpolation methods.

The research-action methodology was followed. Materials, activities and contextualized problems were elaborated and selected, all arranged in the Moodle platform. The statistics generated by the platform provided elements that allowed validating the route. A quantitative analysis was made of the grades and records in the Moodle platform, and a qualitative analysis of the students' perception. The results reveal that the EOS practice system implemented in the Moodle platform improves mathematical learning, achieving 86% school approval. Students recognize the contextualized problems as the element that contributed most to their learning. This proposal could be generalized for teaching computational mathematics in higher education with promising results.

Key words: semiotic, mathematics education, techno-pedagogic, numerical methods, higher education.

Resumo

A disciplina de Métodos Numéricos é obrigatória nos cursos de matemática e engenharia porque é fundamental para muitos desenvolvimentos computacionais atuais. No entanto, embora a sua aprendizagem exija que os alunos tenham conhecimentos matemáticos adequados para compreender e aplicar estes métodos, o ensino normalmente centra-se na mecanização, o que leva a uma aprendizagem insuficiente. Neste contexto, a abordagem ontossemiótica (EOS) (Godino et al., 2007) é um sistema teórico que integra diversas teorias do ensino da matemática e assume que a atividade numérica se concentra na resolução de problemas. Portanto, este estudo se propôs a construir e testar um percurso técnico-pedagógico baseado no sistema de prática EOS implementado no Moodle a fim de desenvolver o conhecimento matemático necessário ao aprendizado e aplicação de métodos de interpolação polinomial. Para isso, seguiu-se a metodologia da pesquisa-ação com o objetivo de desenvolver materiais e atividades, e selecionar problemas contextualizados disponíveis na plataforma Moodle. As estatísticas geradas pela plataforma ajudaram a validar o percurso proposto, sendo realizada uma análise quantitativa das notas e registros no Moodle, juntamente com uma análise qualitativa da percepção dos alunos. Os resultados mostram que o sistema de prática EOS implementado no Moodle melhora a aprendizagem matemática, com uma taxa de aprovação de 86%. Da mesma forma, os alunos destacaram que os problemas contextualizados foram o elemento que mais contribuiu para a sua aprendizagem. Portanto, pode-se sugerir que esta proposta poderia ser generalizada para o ensino de matemática computacional no ensino superior.

Palavras-chave: semiótica, educação matemática, tecnopedagógico, métodos numéricos, ensino superior.

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Introduction

In mathematics, as in most fields of knowledge, proposals, theories and tools constantly emerge that give rise to the emergence of new teaching models. In fact, in the specific case of mathematics, the importance of semiotic functions is recognized, which promote the integration of information and communication technologies (ICT) as tools to improve students' cognitive skills (Sumarwati *et al*.., 2020) and encourage their motivation (Márquez *et al*., 2019).

Taking this perspective into account, the teaching-learning process can be approached from two aspects: the need to master the content and the integration of elements that mark the process mediated using ICT. This approach cannot be arbitrary or disjointed, both technically and pedagogically (Grisales Aguirre, 2018).

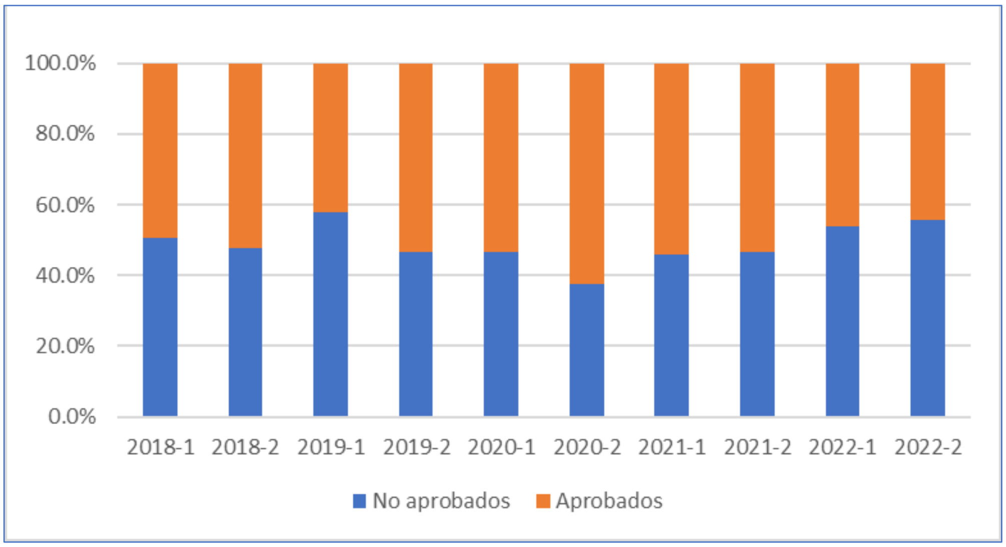
The ontosemiotic approach (EOS) for mathematics instruction (Godino *et al*., 2007) is a theoretical system that integrates various theoretical approaches and models used in research in mathematics didactics. To do this, it establishes criteria for the adequacy between the personal meanings obtained by students (learning) and the intended institutional meanings (teaching), therefore it takes into account the circumstances and resources available in the educational environment (Breda *et al*., 2017). Furthermore, it provides the bases for an educational-instructional proposal that recognizes the importance of the transmission of contextualized and meaningful knowledge for students in the teaching-learning process (Godino, 2019), which implies the participation of the student in the community of practices.

Numerical methods deal with the study, analysis and development of algorithms and procedures to obtain numerical and computational solutions to problems expressed mathematically. These represent a reflection on the analytical methods of algebra and calculus to obtain numerical results of mathematical problems that correspond to various phenomena or processes. Currently, numerical methods are considered essential in scientific computing, which is why they have become fundamental tools for professionals in science, technology, engineering and mathematics careers (Chapra and Canale, 2015).

However, the current teaching of numerical methods faces the main difficulty of linking mathematical knowledge with real-world problems, which is a result of deficiencies in prior mathematical knowledge (Bhatti, 2019) and attitudinal or cognitive problems (Tupacyupanqui-Jaen *et al*., 2018). Therefore, it is common for learning to be reduced to mechanization or memorization, which minimizes mathematical understanding (Montero *et al*., 2015).

For this reason, this work presents a techno-pedagogical route supported by the EOS for the teaching-learning of numerical methods in order to verify its suitability. In the degree in Applied Mathematics and Computing at the National Autonomous University of Mexico (UNAM), two numerical methods courses are taught, in the 3rd and 4th semester, where failure rates are high, as shown in Figure 1. Furthermore, approximately 50% of the non-passed students obtained NP (no show), that is, they dropped out of the course.

Figure 1. Semester pass-fail percentage in numerical methods courses



Source: Coordination of the Applied Mathematics and Computing Program

Given this problem, the thematic unit corresponding to interpolation and polynomial approximation was chosen to implement a techno-pedagogical route supported by the EOS practice system on the course's Moodle platform. The objective of this work was to build and test a techno-pedagogical route based on the ontosemiotic approach implemented in Moodle with the purpose of developing the mathematical knowledge necessary for the learning and application of polynomial interpolation methods.

Literature Review

The teaching-learning of mathematics is not a simple process, as it requires the development of cognitive skills such as observation, analysis and interpretation, among others. Therefore, strategies are necessary that facilitate the acquisition of content through contextualized activities where mental skills are revealed for the construction of new knowledge (Bolaño-Muñoz, 2020).

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According to Duval (2016), semiotic representations are common tools to produce new knowledge and are the only means of access to mathematical objects, while mathematical strategies involve the transformation of representations. This cognitive activity is carried out by those who learn but must be promoted by those who teach. Therefore, the teaching of mathematics demands from the teacher the recognition of the importance of semiotic representations and the mastery of strategies that facilitate these functions.

On this topic, Godino (2011) points out that "technology is essential in the teaching-learning of mathematics, as a means to positively influence what is taught and increase student learning" (p. 13). In addition, technology can be used as a tool to facilitate autonomous and collaborative work, strengthen metacognitive skills and promote social interaction and collaborative problem solving (Galindo Illanes *et al.*, 2022). In this regard, Grisales Aguirre (2018) explains:

> To ensure that the technological tools that are involved in mathematical instruction processes have the desired effects, it is required that the design, implementation and evaluation of resources and strategies be carried out in a rigorous and structured manner within the disciplinary framework. the pedagogical and the technical (p. 210).

The system of practices in the ontosemiotic approach

The ontosemiotic approach (EOS) (Godino *et al*., 2007) is a theoretical system that integrates various theoretical approaches and models used in research in mathematics education. This considers that mathematics is a human activity focused on solving problems through a sequence of practices that involve processes of meaning, conjecture and argumentation. To this end, the notion of problem situation in mathematical practice is proposed (Galindo Illanes *et al*., 2022).

The EOS divides the analysis of the teaching and learning of mathematics into epistemological, cognitive and instructional (Alvarado and Batanero, 2008), which it addresses from the models of meanings of mathematical objects, semiotic functions and didactic configurations. (Godino *et al*., 2007), respectively. Regarding this terminology, it should be noted that a mathematical object is any entity involved in mathematical practice that can be identified as a unit. Therefore, the EOS assumes that the knowledge of an object by a subject (individual or institution) is constructed through a set of semiotic functions, where the object intervenes as expression or content. Finally, the meaning of a mathematical object lies in the correspondence between that object and the system of practices where it intervenes (Galindo Illanes *et al*., 2022).

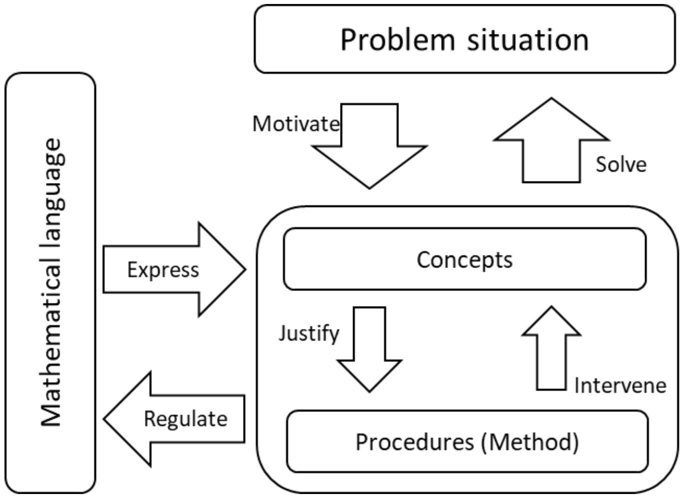
On the other hand, a mathematical practice is defined as a sequence of actions regulated by institutionally established rules, oriented towards an objective, generally solving a problem, communicating the solution to others and validating or generalizing it. For Godino *et al.* (2007), a decomposition of the systems of practices into simpler ones shows the need for objects that allow mathematical activity to be analysed in detail as a relationship of elementary objects, that is, the semiotic function. In this sense, the ontosemiotic configuration represents the relationship between objects, mathematical processes and practices (Pino-Fan, 2017).

Now, the system of practices as a configuration of the EOS consists of dimensions that must be present in the mathematical instruction process (Pino-Fan, 2017):

- Epistemic: The mathematical content studied.
- Cognitive: Prior knowledge and the development of personal meanings.
- Affective: The distribution of attitudes, emotions and motivations.
- Mediational: The (technological) resources used.
- Interactional: The interactions between the participants in the process.
- Ecological: The location of the topic, its relationship with other topics and with the curriculum.

Based on the above, in this work the epistemic configuration proposed by D'Amore *et al*. (2007) was adapted in Figure 2 to relate the mathematical objects involved in the polynomial interpolation methods and the problem situations in a system of practices adapted for implementation on the Moodle platform.

Figure 2. Epistemic configuration of the system of practices



Source: Prepared from D'Amore *et al* (2007, p. 59)

Teaching-learning of numerical methods

Numerical methods constitute a mathematical-computational alternative to solve complex mathematical models whose analytical solution is difficult or impossible to obtain. Although numerical methods courses do not have the main objective of formally demonstrating their mathematical basis, as it is assumed that students already possess it, it is crucial that this is true, given that numerical methods require the computational implementation of mathematical concepts and procedures (Jerše and Lokar, 2017).

This procedural nature leads to students, in addition to facing difficulties in understanding and applying algorithms, reducing their learning to mechanization and memorization processes, which decreases logical reasoning, creative and critical thinking, as well as the ability to search for solutions. and process and analyze information (Montero *et al.*, 2015). Flórez *et al.* (2019) suggest that these difficulties may be the result of the lack of skills and knowledge necessary to address applied problems, where concepts, theorems, and proofs may remain at a level of abstraction that is difficult to assimilate and apply. Furthermore, when solving numerical problems, there is no certainty provided by demonstrations and theorems, which implies living with error and different degrees of uncertainty.

These difficulties, together with the difficulty in linking mathematical knowledge with the real world, represent the main obstacles to the effective learning of numerical methods (Monteiro *et al.*, 2021; Montero *et al.*, 2015). Therefore, in the search for improvements in its teaching, both in its theoretical and practical basis, the use of technologies is resorted to, such as MS Excel tools (Mendonca *et al.*, 2016), Matlab (Monteiro *et al.*, 2021; Rumbaut Leon and Quindemil Torrijo, 2017) or GeoGebra (Allan *et al.*, 2017; Becerra-Romero *et al.*, 2019) to facilitate the numerical-procedural part. These allow the development of practical skills and the active exploration of mathematical concepts, thereby promoting mathematical activity and its application in solving problems (Mendonca *et al*., 2016).

On the other hand, these technologies are also used for pedagogical purposes, creating an environment of exploration that encourages an active role of the student in their learning (Handayani *et al.*, 2017; Rabi and Caneppele, 2018; Raichman *et al.*, 2013), Because in all cases learning through practice is emphasized, which facilitates a deeper understanding and more effective application of numerical methods. In particular, programming stands out as one of the most used resources in teaching numerical methods, since it allows the development of analysis, abstraction and information processing skills (Gwynllyw *et al*., 2020; Jerše and Lokar, 2017).

In addition to these strategies, approaches have been developed that consider attitudinal and cognitive aspects (Montero *et al.*, 2015; Tupacyupanqui-Jaen *et al.*, 2018), such as the flipped classroom model (Clark *et al.*, 2018; Johnston, 2017). Proposals have also been implemented that focus strategies on the student through the use of learning platforms such as Moodle (Becerra-Romero *et al*., 2019), which—according to Handayanto *et al.* (2018)—increase student motivation. Therefore, according to Flórez Escobar *et al.* (2019), contextualized practices are fundamental for learning numerical methods, although it should be noted that none of these approaches explicitly mention the use of mathematics didactics for the design of instructional processes.

Considering that learning is a process influenced by a variety of elements of the subject who teaches, the learner and the learning environment, it is up to the teacher to select appropriate strategies to contribute to the development of skills that facilitate the achievement of the desired learning. The incorporation of technologies for pedagogical and semiotic purposes in the teaching-learning processes of numerical methods provides tools to diversify teaching resources (graphs, animations, tables, diagrams, etc.), which can promote the construction of representations. of the mathematical objects involved.

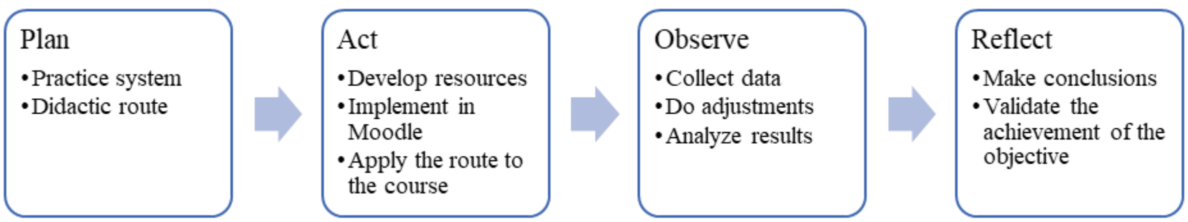
However, it is important to highlight that learning does not depend on the medium used, but on the strategies and techniques applied to that medium, which must promote the student's interaction and communication skills (Tupacyupanqui-Jaen *et al*., 2018). Therefore, in this work a techno-pedagogical route for the teaching-learning of interpolation and polynomial approximation methods is developed and tested.

Method and material

To achieve the objective of this work, the action research methodology was followed with a mixed approach, since an effort was made to collect and analyze results throughout the application of the system of practices for teaching-learning of interpolation and polynomial adjustment methods. The stages of the methodology are shown in Figure 3.

The techno-pedagogical route was applied during the 2023-2 semester, in the Numerical Methods II course of the degree in Applied Mathematics and Computing at the National Autonomous University of Mexico. The study population consisted of two groups with a total of 88 students.

Figure 3. Action research methodology



Source: self-made

The unit corresponding to interpolation and polynomial approximation contemplates the following methods:

- Lagrange interpolation.
- Interpolation by divided differences.
- Interpolation by Newton differences.
- Hermite interpolation.
- Polynomial fit by cubic spline.
- Polynomial fit by least squares.

To evaluate the partial results, the records of the questionnaires generated by the Moodle platform, average grades, consistency coefficient, error rate and standard error were considered (Moodle, 2022), as well as the academic performance at the end of the thematic unit and the students' perception through a survey.

Planning

In the resources and activities for the route, actions corresponding to the dimensions of the EOS practice system shown in Table 1. It is important to clarify that these dimensions are not sequential, that is, they are implicit in the different activities and resources. Likewise, an action can be part of more than one dimension.

Table 1. Dimensions of the internship system

Dimension	Actions
Epistemic	Recover previous knowledge. Promote the construction of necessary knowledge.
Cognitive	Promote mathematical skills. Define procedures. Solve exercises and contextualized problems.
Affective	Generate interest in the topic studied. Motivate participation in different activities. Promote mathematical argumentation.
Interactional	Use the platform as a virtual classroom. Promote teacher-student communication and between students.
Mediational	Use tools to diversify resource formats. Use the Moodle platform for interaction, communication, provision of materials and carrying out activities. Use mathematical *software* and digital resources to explain, visualize and experiment with concepts and procedures.

Source: self-made

In the cognitive and epistemic dimensions, the main objective of the use of technologies is to promote mathematical activity and facilitate abstraction processes (justify-intervene). In the interactional and affective dimensions, technology is used to facilitate interaction between participants (express-regulate) and to motivate the student. Table 2 presents the stages of the technopedagogical route, the result of the adaptation of the epistemic configuration of the system of practices in figure 2 with its corresponding actions and materials. All materials and activities were arranged on the Moodle platform according to the stages of the route as shown in table 2.

Table 2. Stages of the technopedagogical route

Stage	Actions	Materials
Initial presentation	Recover prior knowledge Generate interest	Slides /video /document/graphs
Practical presentation	Promote mathematical skills (apply concepts) Deduce the method Define procedures	Mathematical software (GeoGebra, Mathematica), spreadsheet
Self-appraisal	Solve problems Apply concepts and procedures	Moodle Quiz
Computational implementation	Translate mathematical language into an algorithm	Mathematical software Programming
Application	Resolve problem situations in the internship system	Contextualized problems
Closing	Restructuring of concepts Summative evaluation	Conceptual map

Source: self-made

Finally, the practice system was made up of problem situations (Figure 2) that require interpolation and polynomial approximation methods for their solution. These problems (Table 3) were selected, ensuring that they were as similar as possible to those that the student would encounter in professional practice.

Table 3. Problem situations of the internship system

Method	Problem situation
Lagrange polynomial	Annual divorces in the state of Tlaxcala.
Divided difference polynomial	Daily deaths from covid-19 during the first two months of 2022.
Newton polynomial	Vapor pressures at different temperatures of 1-3 butadiene.
Hermite polynomial	Distance and speed with respect to the travel time of a car.
Fitting by cubic *splines*	Drawing a cartoon profile with GeoGebra.
Least squares adjustment	Construction of models of covid-19 infections by sex and age.

Source: self-made

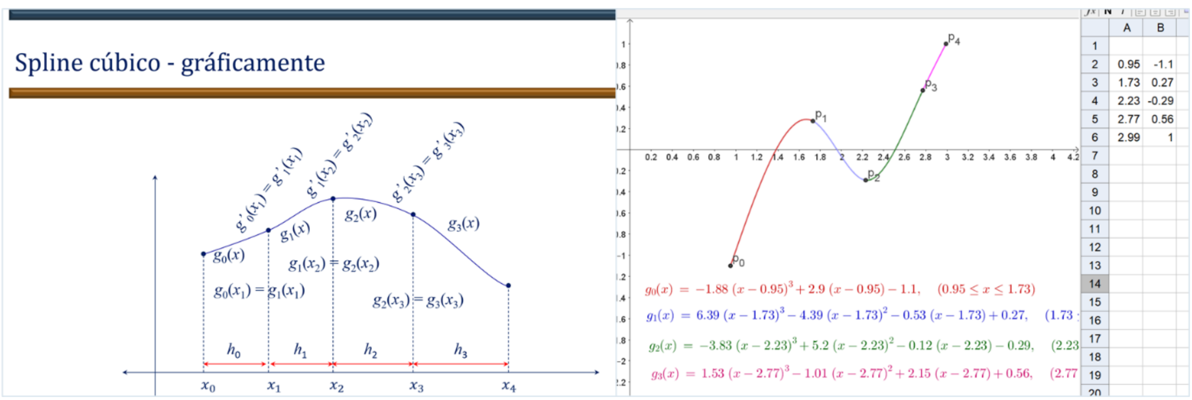
Materials and implementation

For the initial and practical presentations of each method, the materials were prepared according to the epistemic and cognitive dimensions. The content of each one was the following:

- Triggering problem to attract the student's attention.
- Introduction of concepts to recover previous knowledge.
- Graphic supports to the greatest extent possible.
- Deduction of methods with the support of GeoGebra, Mathematica or Excel, applying the concepts and guiding the student in the process.

Examples of cubic *spline* method resources are shown in Figure 4

Figure 4. Examples of resources used for the cubic *spline method*



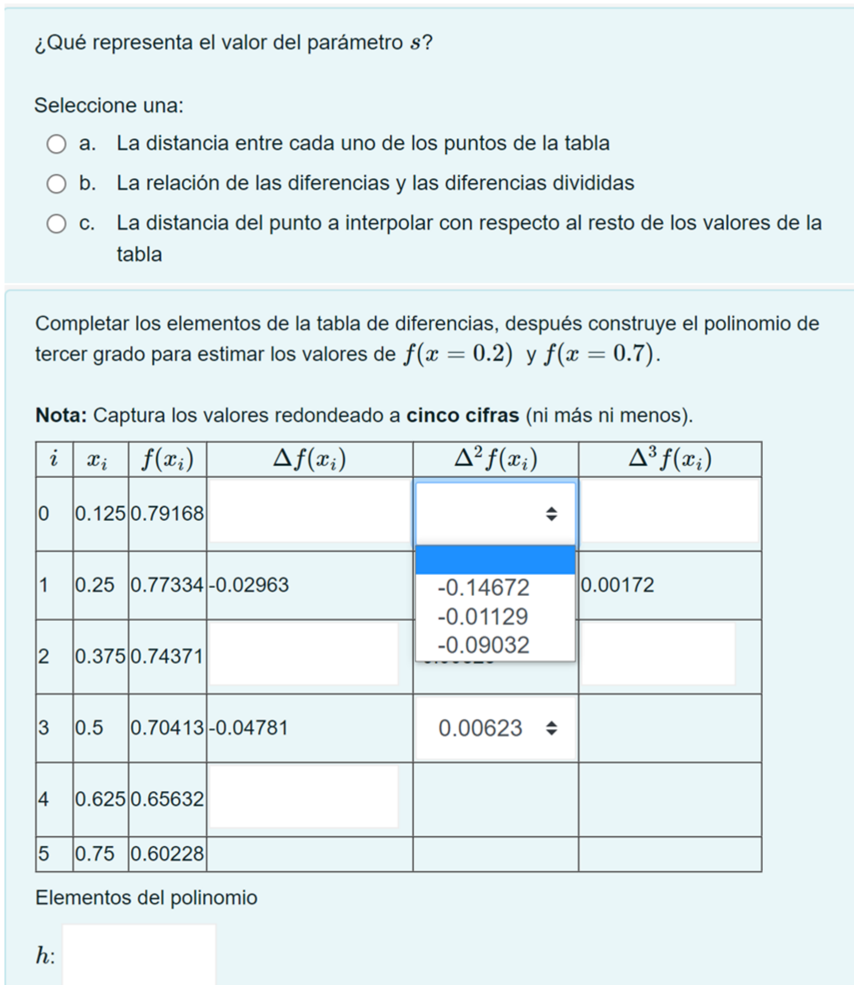
Source: self-made

In addition, through the platform, the resources used in class were provided, such as notes in a PDF document, a video explaining the method and, in some cases, previously validated resources available on the web.

For the self-assessment activities, questionnaires were developed in Moodle, ensuring that the items promoted semiotic functions and the construction of meanings that allowed the student to reaffirm what they had learned and evaluate their understanding of concepts, mastery of the procedure and application of knowledge. In order to motivate the student, three attempts were assigned for each quiz, and the highest score obtained was retained, which would provide the student with the opportunity to review materials and reflect on the answers.

At the end of the questionnaire, as feedback, the student could review the correct answers and understand the main causes of error, which reinforces the cognitive dimension. Examples of the reagents used in these activities are presented in Figure 5.

Figure 5. Examples of items in self-assessment questionnaires


¿Qué representa el valor del parámetro s?

Seleccione una:

- a. La distancia entre cada uno de los puntos de la tabla
- b. La relación de las diferencias y las diferencias divididas
- c. La distancia del punto a interpolar con respecto al resto de los valores de la tabla

Completar los elementos de la tabla de diferencias, después construye el polinomio de tercer grado para estimar los valores de $f(x = 0.2)$ y $f(x = 0.7)$.

Nota: Captura los valores redondeado a **cinco cifras** (ni más ni menos).

i	x_i	$f(x_i)$	$\Delta f(x_i)$	$\Delta^2 f(x_i)$	$\Delta^3 f(x_i)$
0	0.125	0.79168		-0.14672 -0.01129 -0.09032	
1	0.25	0.77334	-0.02963		0.00172
2	0.375	0.74371		[illegible]	
3	0.5	0.70413	-0.04781	0.00623	
4	0.625	0.65632			
5	0.75	0.60228			

Elementos del polinomio

h:

Source: self-made

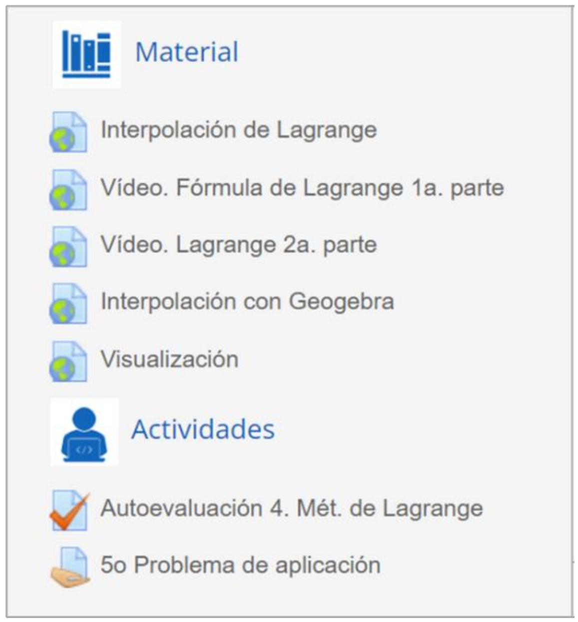
The central activity of the techno-pedagogical route consisted of solving a problem situation—or contextualized problem—of the internship system, in a collaborative way. This activity had to be presented in a formal document for each method with the following structure:

- Analysis of the problem situation.
- Data identification.
- Resolution of the problem through the computational implementation of the method.

- Interpretation of results.
- Argumentation and justification of procedures and results.

The resources and activities described above were provided in a course on the institution's Moodle platform (https://sea.acatlan.unam.mx/), dividing each topic into “Material” and “Activities” as seen in the Figure 6.

Figure 6. Implementation of the technopedagogical route in Moodle



Source: self-made

At the end of the thematic unit, as closing activities, the student created a concept map as an instrument for reaffirming or restructuring knowledge and as preparation for the unit evaluation. With this the route was concluded.

Results

Self-assessment questionnaires represented a means for students to identify progress in their learning. Furthermore, as a formative assessment, they contributed to self-regulation and autonomous learning. Table 4 shows the average scores obtained in the self-assessment of each method and in the solution of the problem situation. As can be seen, the average grade in the last attempt was greater than 8/10 in all methods, which suggests that the activity significantly supported the student, who largely achieved the expected learning. This learning is reflected in the results obtained in solving the problems of the internship system, where in all cases the grades were higher than those obtained in the self-assessment.

Table 4. Average grades obtained in the activities of the route

Method	average mark		
	Self appraisal		Internship system
	First attempt	Last chance	Problem situation
Lagrange polynomial	7.61	8.55	9.34
Divided difference polynomial	7.38	8.23	8.40
Newton polynomial	8.52	9.30	9.62
Hermite polynomial	7.83	8.43	9.30
Fitting by cubic *splines*	8.03	8.84	9.10
Least squares adjustment	7.82	8.50	8.77

Note: The activities were delivered by different numbers of students (Table 6).

Source: self-made

The self-assessment questionnaires were validated using the records and statistics generated by the Moodle platform:

- Internal consistency indicates whether the questions discriminate between students with different abilities. A value greater than 70% is considered satisfactory, while a value less than 64% indicates unsatisfactory.
- The error rate estimates the percentage of the standard deviation that is due to random effects rather than genuine differences in ability between students. Values greater than 50% are not satisfactory.
- The standard error is a measure of uncertainty in any student's grade. If the standard error exceeds 8%, it is likely that a substantial proportion of students are graded incorrectly.

These statistics are concentrated in Table 5It can be seen that the Newton polynomial questionnaire was the least satisfactory; In addition, a problem is detected in the standard error, which indicates that an analysis is required on the factors that may be affecting the students' responses, such as time, the platform, the reagents or the equipment on which they are solved. to name a few.

Table 5. Questionnaire statistics

Method	Internal consistency	Error rate	Standard error
Lagrange polynomial	81.91	42.54	7.53
Divided difference polynomial	77.81	47.10	9.30
Newton polynomial	72.73	52.52	8.79
Hermite polynomial	78.34	46.54	11.38
Fitting by cubic *splines*	80.58	44.05	6.94
Least squares adjustment	70.16	46.43	7.18

Source: self-made

The results regarding student participation in the activities of the techno-pedagogical route (table 6) reveal the following:

- Student participation decreased with progress in the unit.
- *spline* adjustment method is the one that had the least participation.
- Making additional attempts to improve the grade on the self-assessment activity also decreased with progress in the unit.
- Participation in solving the problems of the practice system was less than in the self-evaluation in all cases.

Table 6. Student participation in activities

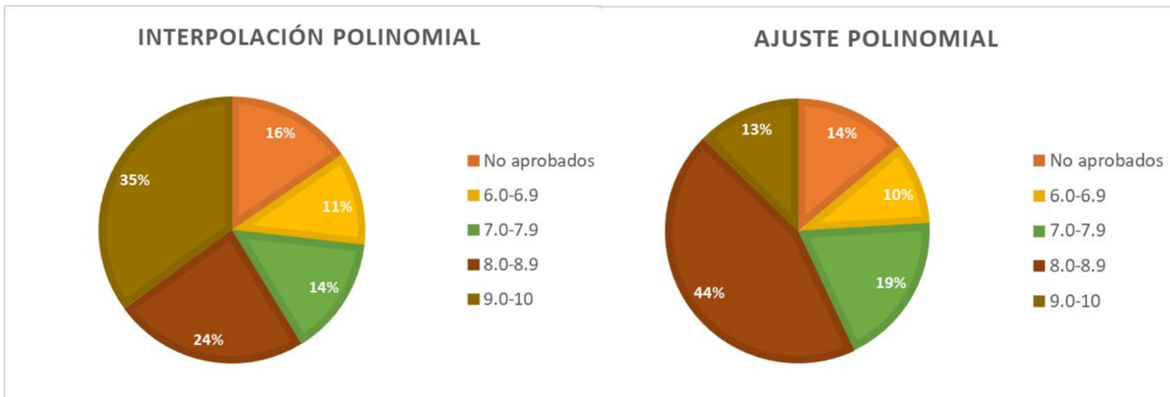
Method	Self appraisal		Problem situation
	Participants	Total Attempts	Deliveries
Lagrange polynomial	88	159	69
Divided difference polynomial	87	158	66
Newton polynomial	79	124	63
Hermite polynomial	77	127	57
Cubic *spline* fitting	69	100	fifty
Least squares adjustment	73	109	55

Source: self-made

Subsequently, to evaluate the results of the practice system as a whole, the student's performance was considered through their grade at the end of the thematic unit, divided into two blocks: interpolation polynomials and polynomial adjustment. In this sense, it is suggested that the use of the EOS practice system implemented in Moodle as a techno-pedagogical route generated satisfactory results.

In Figure 7 it can be seen that only 15% of the students did not achieve the minimum learning required to pass, while more than 50% obtained a grade higher than 8/10. Considering that historically the failure percentage is usually around 60% and the average grade obtained was 7.5/10, it is suggested that the use of the EOS practice system implemented in Moodle as a techno-pedagogical route generated satisfactory results.

Figure 7. Distribution of final grades by subject reviewed



Source: self-made

Finally, for the qualitative analysis, a survey was applied to know the students' opinions, which was answered by 78 of them, of which 74% considered that the contextualized problems facilitated the understanding of the method (figure 8) and were the component of the route that contributed the most to their learning (figure 9).

Given both questions, it is observed that students have preference for class sessions and for activities that require their active participation, such as problems, partial evaluations and self-assessment activities. This is confirmed by their opinion about concept maps, that despite their purpose being to integrate and reaffirm knowledge to prepare them for the evaluation, students did not perceive it as an important activity for their learning (figure 9).

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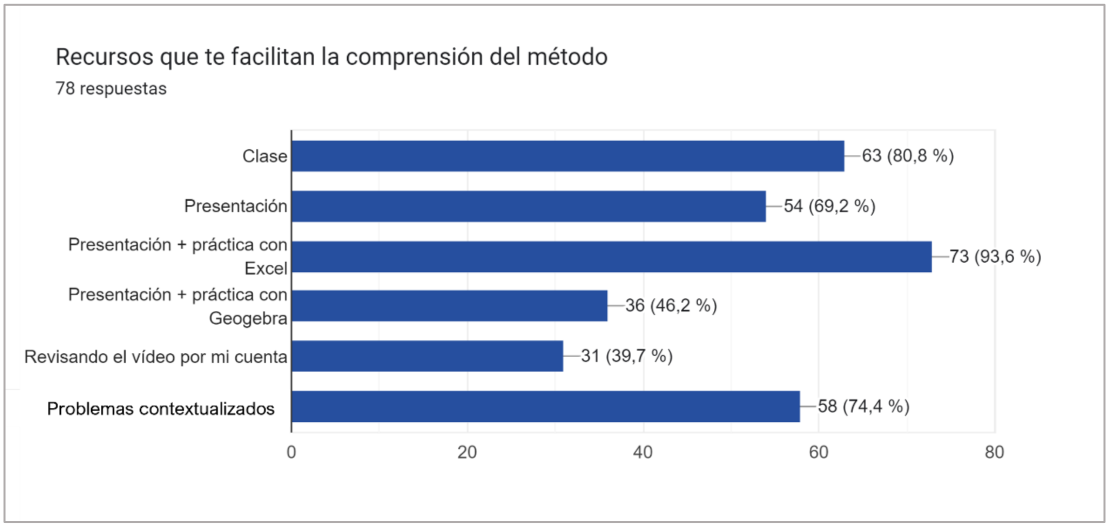
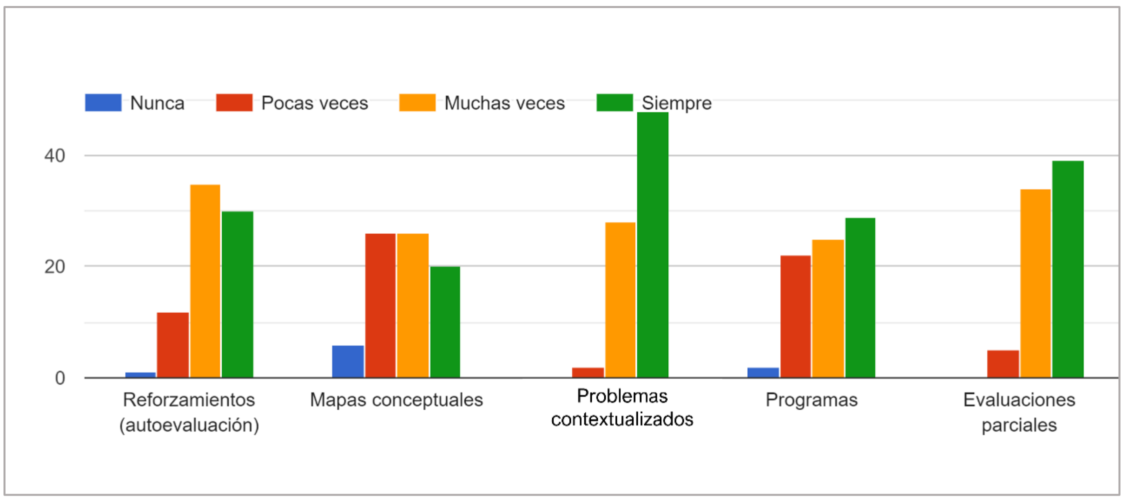
Figure 8. Perception of learning resources of the technopedagogical route



Source: self-made

Figure 9. Elements of the technopedagogical route that contribute to learning



Source: self-made

Discussion

In this work, a techno-pedagogical route based on the EOS practice system was designed, built, implemented and evaluated in order to promote the mathematical knowledge required for the learning and application of polynomial interpolation methods. In this sense, it is observed that the route is feasible and, in general, seems beneficial for the achievement of learning.

One of the main findings was the students' perception of the internship system. Despite representing a considerable workload during the semester, they recognize that solving contextualized problems is the main element to achieve learning. This allows us to affirm that the system of practices as a configuration of the ontosemiotic approach, appropriately designed, can be adapted to the Moodle platform with good acceptance by students. In other words, the main assumption of the EOS that mathematical activity is a human activity focused on problem solving is met.

For its part, the diversity of formats in the resources and activities of the techno-pedagogical route, using ICT, seems to promote semiotic transformations, essential to produce new knowledge (Duval, 2016), whenever technology is used to influence positively in what is taught (Godino, 2011). In this sense, it should be noted that the epistemic, cognitive, affective, mediational and interactional dimensions of the system of practices were integrated into materials and activities. In fact, in all cases ICTs were used to promote semiotic functions, facilitate interaction and promote motivation.

Likewise, they were implemented as a collaboration tool and to perform mathematical operations, which constitutes a substantial part of the techno-pedagogical route. Thus, it seems imperative to integrate ICT into the teaching-learning processes of numerical methods, especially now that, since the confinement derived from covid-19, technology has become an important part of didactics.

Regarding teaching activities, the survey reveals that students identify practical activities as important elements for their learning, so the teacher must carefully design them to achieve the necessary theoretical learning that allows a correct application of concepts and methods (D 'Amore *et al.*, 2007). Likewise, the problems integrated into the internship system must be close to the students' reality and be continually updated so that they resemble those that graduates will face in their profession. However, this requires the teacher not only to have a deep mastery of the contents, but also to have detailed knowledge of the EOS, specifically of the internship system, which represents the main difficulty of the proposal.

On the other hand, the development of evaluation activities represents a considerable demand of time and academic work for teachers. Even so, the statistics generated by the Moodle platform are a valuable tool to validate the quality of the evaluation and to facilitate the monitoring of student performance and the construction of their knowledge. Furthermore, developing interactive activities in Moodle is profitable for the teacher, since, although it requires initial effort, the activities become assets that can be improved each semester. That is, the teacher can reuse and continually improve them.

Now, although some students do not see the importance of this subject in their professional life (Montero *et al.*, 2015) and sometimes do not consider it specifically mathematics (Flórez Escobar *et al.*, 2019), it must be emphasized that incorporating the ICT in the internship system favors the construction of mathematical knowledge. In this regard, it was evident that students prefer teaching activities that require their direct participation, which had a positive influence on the learning of numerical methods.

In summary, it can be assured that the techno-pedagogical route shown, based on the EOS system of practices, is seen as promising and feasible.

Conclusions

In conclusion, using the EOS practice system in the teaching-learning of numerical methods favors the construction of students' mathematical knowledge, since it improves the learning of methods such as polynomial interpolation. Specifically, the contextualized problems of the practice system are the element that contributes the most to learning, according to the perception of the students (figures 8 and 9), which suggests that the objective of the practice system of promoting effective learning through problem solving.

Regarding the techno-pedagogical route, given that the grades obtained in the problems of the practice system are higher than those achieved in the self-assessment activities, it can be assumed that the stages of the route follow an appropriate sequence for the student to build knowledge. necessary to solve problems effectively.

Finally, the possibilities offered by the Moodle platform for the implementation of EOS configurations suggest that it could be adapted to a virtual course, which would take advantage of the technological tools available to improve the students' learning experience.

Future lines of research

Based on the results obtained in this work, it can be indicated that the main future challenge would be to establish a comprehensive model that allows teachers to integrate EOS into their teaching in a simple and practical way. This model, which was initially proposed for a thematic unit, could be extended to other and similar subjects of computational mathematics.

On the other hand, a decrease was observed in student participation in activities throughout the thematic unit (table 6). Although this phenomenon is common, it should be considered for further research that addresses motivational and attitudinal factors.

In this proposal, the Moodle platform was used as a support resource for face-to-face learning, which greatly facilitated the implementation of EOS. However, it would be interesting to expand the proposal to a completely virtual modality. In addition, the EOS contemplates other configurations that could be incorporated to improve results, even in computational mathematics subjects in higher education, which are not exclusively mathematics but rather applications of them.

The techno-pedagogical route presented here, implemented on the Moodle platform, is a concrete example of a practical application of EOS, which has generated satisfactory results in terms of qualifications and acquisition and application of mathematical knowledge. Therefore, this work represents an invitation to continue exploring the applications of EOS as an innovative strategy.

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Contribution Role	Author(s)
Conceptualization	Teresa Carrillo Ramirez
Methodology	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports"
Software	Does not apply
Validation	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports" Sandra Luz Canchola Magdaleno "who supports"
Formal Analysis	Teresa Carrillo Ramírez "same" María del Carmen González Videgaray "same"
Investigation	Teresa Carrillo Ramirez
Resources	Teresa Carrillo Ramirez
Data curation	Teresa Carrillo Ramirez
Writing - Preparation of the original draft	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports"
Writing - Review and editing	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports" Sandra Luz Canchola Magdaleno "who supports"
Display	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports" Sandra Luz Canchola Magdaleno "who supports"
Supervision	Teresa Carrillo Ramírez "main" María del Carmen González Videgaray "who supports"
Project management	Teresa Carrillo Ramirez
Fund acquisition	María Teresa García Ramírez