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Artículos Científicos

La comprensión de conceptos fundamentales del cálculo mediante Desmos. Una intervención

Understanding Fundamental Concepts of Calculus Through Desmos. An Intervention

A compreensão de conceitos fundamentais de cálculo usando Desmos. Uma intervenção

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Resumen

En este artículo se realiza el diseño de una estrategia metodológica para la comprensión del concepto de *derivada*. Esto a través de Desmos y el programa "trae tu propio dispositivo" (BYOD, por sus siglas en inglés) en un primer curso de cálculo diferencial a nivel licenciatura, así como mediante la herramienta metodológica heurística de investigación-acción. Para lograr el objetivo se involucra, además, el método de derivación por incrementos, el cual se codifica para ser incorporado a la plataforma Desmos y con ello realizar animaciones. Posteriormente, se realizan *quizzes* a través de Kahoot! para evaluar su comprensión. Se concluye que Desmos es un auxiliar de utilidad en la comprensión de conceptos fundamentales del cálculo, ya que permite el esbozo de funciones de manera sencilla; así, da sentido a la matematización, aunque no al cálculo de resultados.

Palabras clave: aprendizaje, derivada, interpretación, límite.

Abstract

The design of a methodological strategy for the understanding of the concept of derivative is carried out in this article. This through the Desmos software and the bring your own device (BYOD) program in a first course of differential calculus at the degree level, using the action research methodology. Also, it is involved the method of derivation by increments, which is coded to be incorporated into the Desmos platform and thus make animations. Quizzes are then performed through Kahoot! to evaluate the understanding of the students. It is concluded that Desmos is an useful aid in the understanding of fundamental concepts of calculus, since it allows the sketching of functions in a simple way giving meaning to mathematization, but not the calculation of results.

Keywords: learning, derivative, interpretation, limit.

Resumo

Neste artigo, é realizado o desenho de uma estratégia metodológica para a compreensão do conceito de derivada. Isso através do Desmos e da prática "traga seu próprio dispositivo" (BYOD) em um primeiro curso de cálculo diferencial no nível de graduação, bem como através da ferramenta metodológica heurística da pesquisa-ação. Para atingir o objetivo, também está envolvido o método de derivação incremental, que é codificado para ser incorporado à plataforma Desmos e, assim, fazer animações. Posteriormente, os testes são realizados através do Kahoot! para avaliar sua compreensão. Conclui-se que Desmos é uma ajuda útil na compreensão de conceitos fundamentais de cálculo, pois permite o esboço de funções de maneira simples; Assim, dá sentido à matematização, embora não ao cálculo dos resultados.

Palavras-chave: aprendizagem, derivada, interpretação, limite.

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Introduction

When one talks about the difficulties of imparting differential and integral calculus, one speaks, in part, of overcoming the mechanization of algebraic processes. Limit and derivative are fundamental concepts that the student must understand to develop the ability to calculate and interpret and, in the same way, to incorporate later knowledge into sciences such as physics and mathematics. Learning mathematics not only requires the ability to perform exercises, but also to translate ideas and representations through symbols, which represent a movement, a state, a variation or an unknown factor. To that idea embodied in the form of an equation or operation, if its meaning is not known, it is impossible to add more knowledge, which affects the resolution of a problem. As White and Mitchelmore (1996) point out, the resolution of application problems must be accompanied by conceptual knowledge and not by "instrumental understanding".

In an investigation related to the process / object for the limit case, Cottrill et al. (1996) emphasize that the difficulty in understanding the concept of limit lies in the fact that it requires the reconstruction of two coordinated processes. Later, Sierra, González and López (2000) point out the difficulty of understanding the concepts of limit and continuity even after the teaching process. Whereas Ferrini-Mundy and Graham (1994) documented the student's difficulty in connecting the symbolic representation of a derivative with any type of geometric understanding. And a later investigation, by Habre and Abboud (2006), indicates that, from a teaching with an emphasis on visualization, students understand the derivative as an instantaneous change or as the slope of a curve at a point. For their part, Martínez, López, Gras and Torregrosa (2002) describe the contributions and insufficiencies of the historical conceptions of Leibnitz and Cauchy.

In this line drawn by the examples cited above, this research has shown the poor interpretation of fundamental concepts in calculus and an insufficient ability in algebraic handling in the calculation of limits and integrals. Ferrini-Mundy and Gaudard (1992) also show that a teaching based on algorithmic processes is inadequate to achieve the correct interpretation of concepts within differential calculus. Likewise, Rojas (2015) documents didactic sequences that seek to promote the understanding of the concept of the limit of a function, which rest on an adequate understanding of the concept, which was not the case in the early 19th century with Cauchy.

Based on this and knowing the importance of algebraic handling and the resolution of limits and derivatives, a proposal is made through drawing and digital animation to seek to influence the understanding of the concept of derivative with the help of Desmos, a graphing calculator freely accessible and available for various operating systems and devices.

En Rojas (2019), Thomas (2015), Montijo (2017) y Almarshedi, Wanick, Wills y Ranchhod (2017) The importance of drawing is pointed out through the Desmos graphing calculator and the student's attitudinal transition to learn through gamification. However, drawing by itself does not affect an answer, but rather an order of ideas or steps to develop, or to capture a phenomenon, whether physical or chemical, to name a couple of examples.

However, deriving a function does not imply an understanding of its use; When we talk about a derivative, we are talking about changes, variations, and perhaps it is not interesting to know the results of those changes, but how they originated and what they entail; perhaps they happened in a very short time, which may go unnoticed by the "human eye". So it is necessary to redefine the phenomenon and limit it in a certain interval. However, the derivation by increments allows us to formally define the derivative. This method is also known as the four-step rule, which has the particularity that it is necessary to involve limits; in addition, it allows internalizing learning as opposed to derivation by formulas; or, as a differential, that it came from a "stormy" process for Newton and Leibnitz and that it came to light with the arrival of Cauchy.

> The increment of any continuous function obeys the formal definition of infinitesimal, it does not make sense to use the differential term to refer to the increment (infinitesimal) of a function. If we add to this the suspicion accumulated over the years about the differential and the infinitesimals to serve as the basis for less rigorous mathematical treatments, the terrain was clearly paved so that the differential was relegated to a marginal role in the new theoretical framework calculation (Martínez *et al*., 2002, p. 275).

Once the foundation of the derivative is strong, it is possible to evolve the instruction to calculate the derivative through algorithmic processes.

The importance of knowing the derivation by increments lies in the mathematization and mathematical modeling (Rojas, 2018; Martínez, Cobos and Torres, 2015; Arrieta and Díaz, 2015) for the appropriation of language and the synthesis of an already subsequent conceptualization. In addition, the recovery of knowledge a priori is important to create a mathematical scaffolding.

All of the above constitutes an accumulation of reasons sufficient to carry out an investigation where the incremental derivation method is programmed with the help of the Desmos graphing calculator and analyze its contribution to the understanding of the derivative in a calculation course.

Research Problem

How does using the Desmos graphing calculator help you understand the derivative?

Objective

Program using the graphical calculator Desmos the incremental derivation method

Method

The present work is shaped by the form of action research, since the intervention and analysis of this is what makes us improve our understanding of educational reality and transform it according to the technical and practical modality that characterizes it (Colmenares and Piñero , 2008). There was the participation of 20 students enrolled in the Differential and Integral Calculus course of the 2nd semester of the Bachelor's degree in Biotechnology from the Universidad Michoacana de San Nicolás de Hidalgo. Participants are between the ages of 17-19. The practice "bring your own device" (BYOD) was followed: the students worked with their smartphone and, from there and through an Internet connection, programmed and accessed the Desmos graphing calculator to carry out the activity . After experimenting with the app, the data was collected through Kahoot!, A platform that allows, as a game, to take a quiz and verify the answers instantly; At this time the use of the Desmos app was not allowed. During any intervention, observations of empathy, management and difficulties that were presented during the experiment were taken.

Development

The four-step rule or derivation by the increment method, according to Cauchy, consists by definition of the following:

Be a function $y = f(x)$

1) $y + \Delta y = f(x + \Delta x)$

2) $\Delta y = f(x + \Delta x) - f(x)$

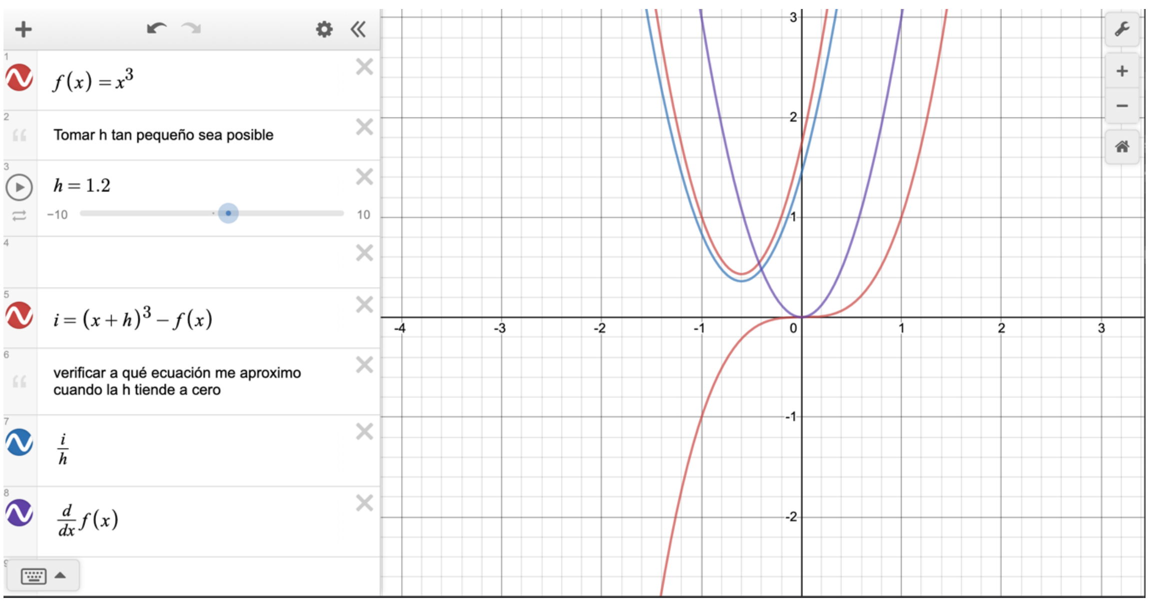
3) $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$

4) $y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

We transfer this process to code; then we program it in the Desmos graphing calculator, defining, for example, the function $f(x) = x^3$ arbitrarily.

We assign a value h as small as possible, and that goes through negative numbers with a step number greater than two decimal places, as shown in figure 1.

Figura 1. Algoritmo del método de derivación por incrementos en Desmos

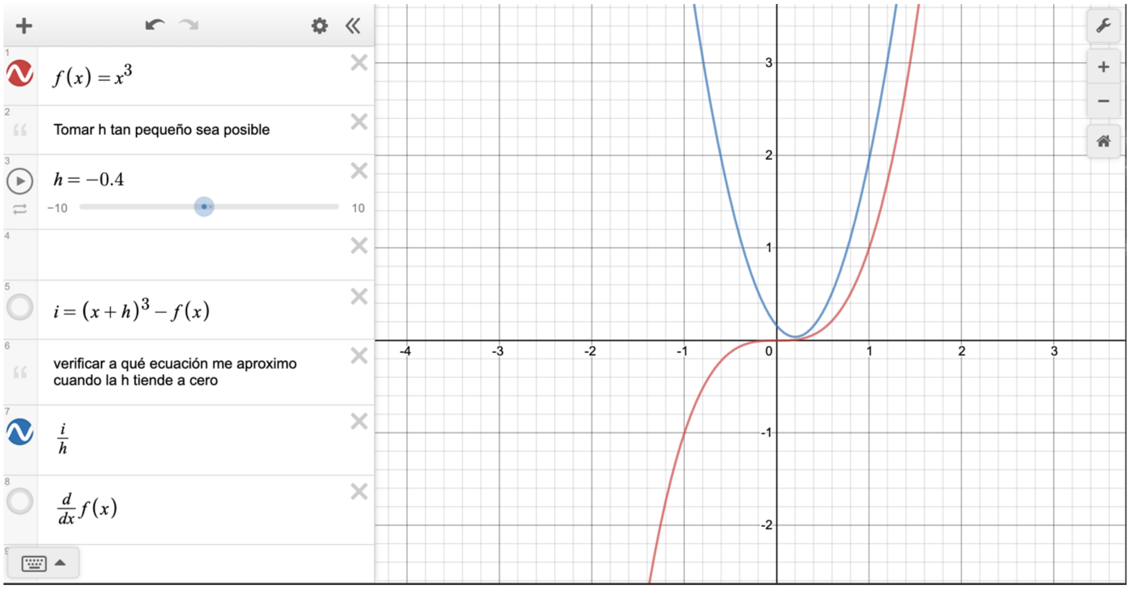


Fuente: Elaboración propia

It is convenient to graph line eight in Desmos to make the comparison of the values previously emitted and to allow comparing the derived function.

Likewise, it is recommended to disable row five and eight, as shown in figure 2.

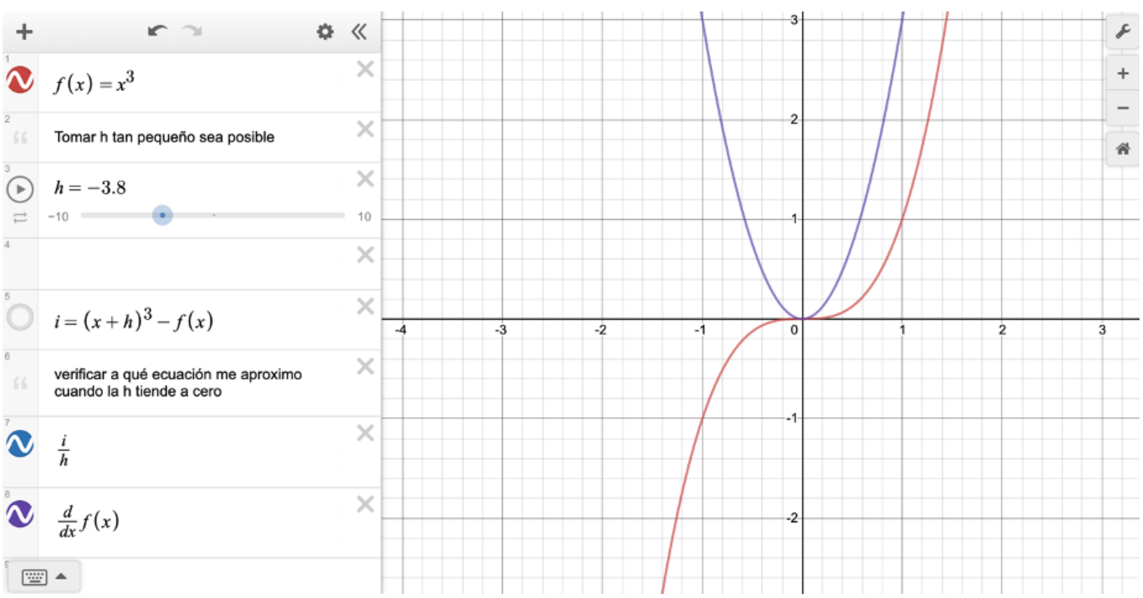
Figura 2. Visualización de la derivada en un punto con Desmos



Fuente: Elaboración propia

Once this has been done, all you have to do is move the slider of h as close to zero as possible $\Delta x \to 0$, and then run the animation.

Figura 3. La derivada en un punto versus el límite de la razón de los incrementos cuando la variable dependiente tiende a cero mediante Desmos



Fuente: Elaboración propia

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The teacher questions the students with the following:

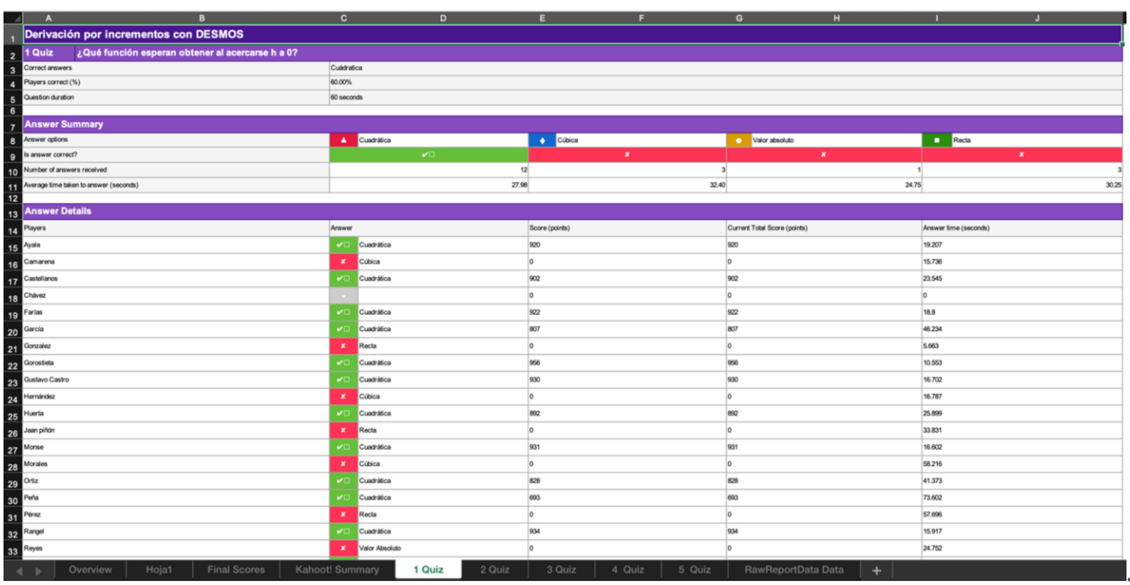
1) What function do you hope to get as h approaches zero?
2) What function is obtained when h = 0?
3) What is the limit of a function?
4) What is the derivative of a function?
5) Is every function differentiable?

Results

After experimentation with the Desmos platform, the previously mentioned questions are asked. The results were captured through five quizzes on the Kahoot! Platform, with a response time of up to 60 seconds, except for Quiz 5 (Q5), which was up to 90 seconds due to the complexity it represents. If the student answered outside of that time interval, the answer was considered neither for the quiz score nor for the statistics involved in this work.

The results found are observed in figure 4 and in table 1.

Figura 4. Matriz de resultados obtenidos de la plataforma Kahoot!



Derivación por incrementos con DESMOS					
1 Quiz	¿Qué función esperan obtener al acercarse h a 0?				
Correct answers		Cuádratica			
Players correct (%)		60.00%			
Question duration		60 seconds			
Answer Summary					
Answer options		Cuadrática	Cúbica	Valor absoluto	Recta
Is answer correct?		✔	✘	✘	✘
Number of answers received		12	3	1	3
Average time taken to answer (seconds)		27.98	32.40	24.75	30.25

Answer Details

Players	Answer		Score (points)	Current Total Score (points)	Answer time (seconds)
Ayala	✔	Cuadrática	920	920	19.207
Camarena	✘	Cúbica	0	0	15.736
Castellanos	✔	Cuadrática	902	902	23.545
Chávez	-		0	0	0
Farías	✔	Cuadrática	922	922	18.8
García	✔	Cuadrática	807	807	46.234
Gonzalez	✘	Recta	0	0	5.663
Gorostieta	✔	Cuadrática	956	956	10.553
Gustavo Castro	✔	Cuadrática	930	930	16.702
Hernández	✘	Cúbica	0	0	16.787
Huerta	✔	Cuadrática	892	892	25.899
Jaan piñón	✘	Recta	0	0	33.831
Monse	✔	Cuadrática	931	931	16.602
Morales	✘	Cúbica	0	0	58.216
Ortiz	✔	Cuadrática	828	828	41.373
Peña	✔	Cuadrática	693	693	73.602
Pérez	✘	Recta	0	0	57.696
Rangel	✔	Cuadrática	934	934	15.917
Reyes	✘	Valor Absoluto	0	0	24.752

Fuente: Elaboración propia

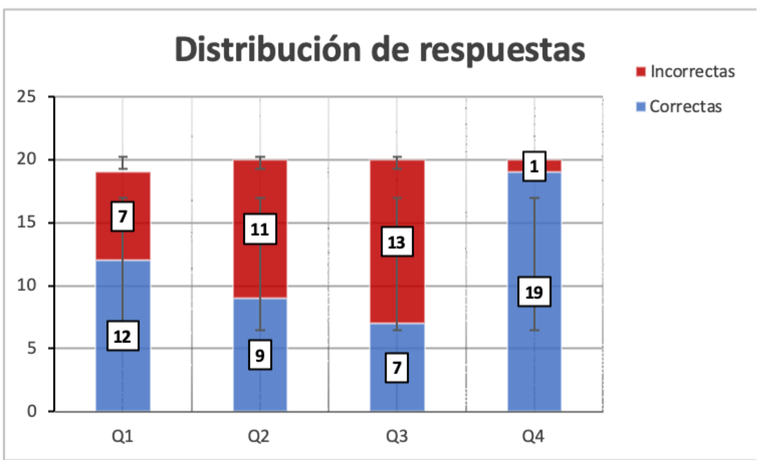
Tabla 1. Distribución de respuestas

	Quiz	Correctas	Incorrectas
Válido	Q1	**12**	**7**
	Q2	**9**	**11**
	Q3	**7**	**13**
	Q4	**19**	**1**
	Q5	**1**	**18**
	Total		**98**

Fuente: Elaboración propia

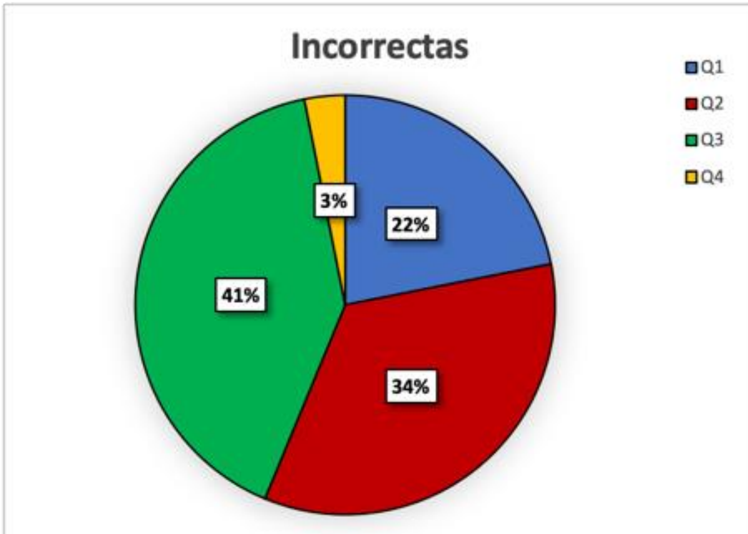
Given the complexity of Q5, it is omitted from the analysis in the following figures, since it requires a particular interpretation of its results; in addition, it does not belong to the objective of the present work; However, it was disturbing to know how far a mathematical analysis could go.

Figura 5. Desviación estándar de la distribución de respuestas por número de *quiz*



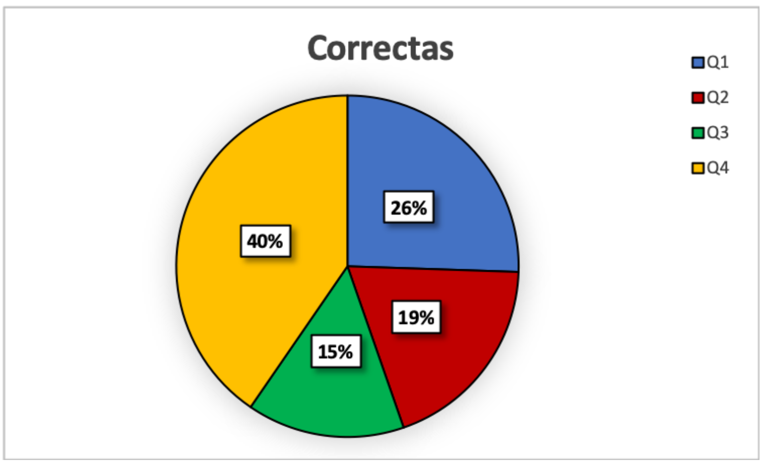
Fuente: Elaboración propia

Figura 6. Porcentaje de respuestas incorrectas por número de *quiz*



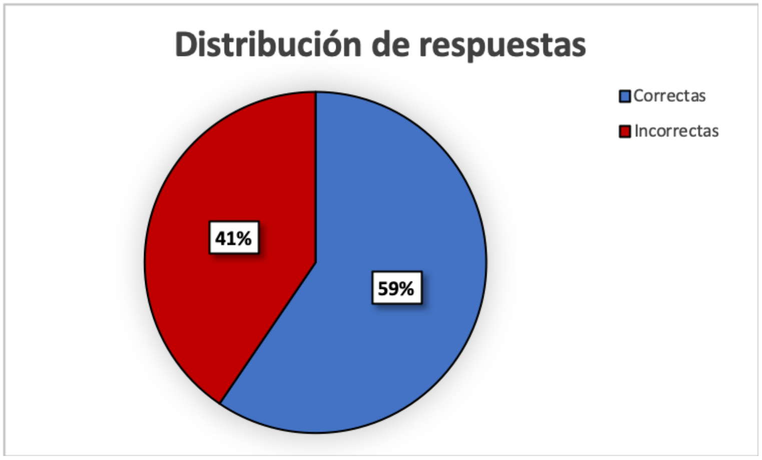
Fuente: Elaboración propia

Figura 7. Porcentaje de respuestas correctas por número de *quiz*



Fuente: Elaboración propia

Figura 8. Distribución de porcentajes totales de respuestas



Fuente: Elaboración propia

Discussion

It has become evident that the use of Desmos in the classroom favors the understanding of the concept of derivative of a function (Q4). However, there are areas of opportunity.

During the observations that were made in the classroom at the time of the intervention with the platform, the students showed confusion when programming. Well, creating a slider was assumed as a point; others defined the function as y, where it is mathematically correct but in the algorithm it is not convenient to write the function as such, but to define it as a function that depends on a variable x. At the time of finishing the programming on the platform and executing the animation, the attitude was not one of empathy or enthusiasm. In addition, the reflection of a learning was not manifested, but rather the obedience to follow the steps that the teacher indicated to carry out the task.

It was also evident that they still cannot understand the definition of limit (Q3), analogously to those raised in Sierra et al. (2000), because, despite having been treated in a previous thematic unit, the students could not relate it: the idea persists that the limit is based on a substitution of the dependent variable (mathematically speaking and not methodologically), regardless of that it is impossible to obtain its result. On the contrary, the identification of functions (Q1) was highly favored at the time of graphing, since they managed to identify, not exactly, but rather intuitively, the function that is expressed mathematically.

The last quiz (Q5) had a degree of difficulty far beyond the limits of the objective in this traced investigation, since to answer it, higher knowledge and an advanced subject, such as mathematical analysis, were required; However, challenging students can be interesting, although it is obvious that they did not respond correctly, but it creates the antecedent that it is not always possible for a function to be differentiable at all points. Although a single student was correct in the answer, he could not justify it.

Limitations

Sample size

Statistical tests usually require a larger sample size to ensure a representative distribution of the population and to be considered representative of the groups of people studied. Although the sample size is less relevant in qualitative research, it is imperative to point out that when we talk about a teaching process, and more in mathematics, it should not be approached before a massive group, despite the fact that some groups are actually like this structured, as in some schools or universities. So if it is a group beyond 20 students, technology in the classroom is not recommended.

Infrastructure

Students must have a relatively new smartphone, where the Desmos platform can be downloaded and updated, as well as having the use of mobile data or a wireless connection.

Platform knowledge

The teacher must have the skills and knowledge of common smartphone operating systems to assist in the execution and distribution of appropriate permissions for the correct installation and management of the Desmos platform.

Conclusions

It is generally concluded that Desmos is a useful auxiliary to the task of sketching a function, since it simplifies the computation and tracing of functions, and indicates the discontinuous points that operationally could be overlooked. But the ability and detection of problem points, as well as the interpretation of the results, remains human terrain, that is, there is no understanding if there is no brain development, a work that challenges it, and this is where the skills individual are externalized. The use of technology in the classroom allows us to model certain realities and understand definitions that can certainly be difficult to understand, but the use of technology in mathematics is not a valid way to prove theorems. Although it is possible to find the derivative of a function through increments, it is not convenient to resort to it as the only procedure. Well, despite being a holistic method that includes a limit, the time factor can play a powerful role, especially if it is a function that needs a lot of algebraic or trigonometric experience to be found in a synthesized way. So it is recommended that the step be taken to calculate the derivative by formulas.

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